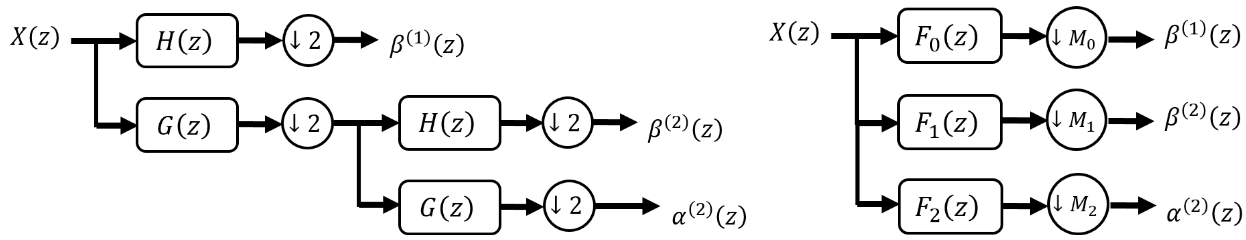
Full Name: ____________________

EEE 5502 (Fall 2022) – Practice **Multi-Resolution Applications, 2022**

Question #1: Consider the following analysis wavelet bank and filter bank.



Let these filters be defined by

$$H(z) = (1/\sqrt{2})(1 - z^{-1})$$
$$G(z) = (1/\sqrt{2})(1 + z^{-1})$$

(a) Use the Noble identities to simplify the analysis wavelet bank (left) diagram and represent it as a filter bank (right). Determine M_0, M_1, and M_2.

Solution:

$$M_0 = 2$$
$$M_1 = 4$$
$$M_2 = 4$$

$$F_0(z) = (1/\sqrt{2})(1 - z^{-1})$$
$$F_1(z) = (1/2)(1 + z^{-1} - z^{-2} - z^{-3})$$
$$F_2(z) = (1/2)(1 + z^{-1} + z^{-2} + z^{-3})$$

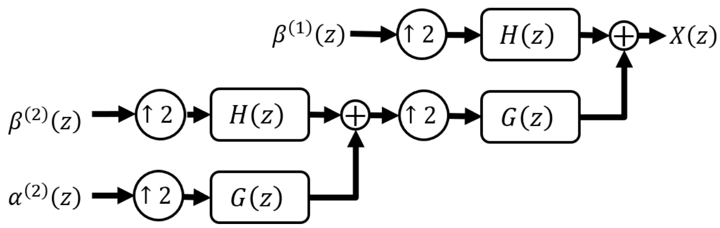
(b) Compute $\beta^{(1)}(z)$, $\beta^{(2)}(z)$, and $\alpha^{(2)}(z)$ for an input $x[n] = \delta[n] + 2\delta[n-1]$

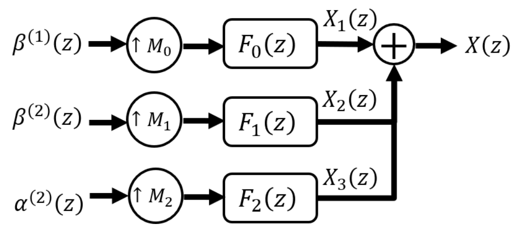
Solution:

$$
\begin{aligned}
X(z)F_0(z) &= (1/\sqrt{2})(1 + z^{-1} - 2z^{-2}) \\
X(z)F_1(z) &= (1/2)(1 + 3z^{-1} + z^{-2} - 3z^{-3} - 2z^{-4}) \\
X(z)F_2(z) &= (1/2)(1 + 3z^{-1} + 3z^{-2} + 3z^{-3} + 2z^{-4})
\end{aligned}
$$

$$
\begin{aligned}
\beta^{(1)}(z) &= (1/\sqrt{2})(1 - 2z^{-1}) \\
\beta^{(2)}(z) &= (1/2)(1 - 2z^{-1}) \\
\alpha^{(2)}(z) &= (1/2)(1 + 2z^{-1})
\end{aligned}
$$

Question #2: Consider the following synthesis wavelet bank and filter bank.





Let these filters be defined by

$$H(z) = (1/\sqrt{2})(-z^{+1} + 1)$$
$$G(z) = (1/\sqrt{2})(z^{+1} + 1)$$

(a) Use the Noble identities to simplify the analysis wavelet bank (left) diagram and represent it as a filter bank (right). Determine M_0, M_1, and M_2.

Solution:

$$M_0 = 2$$
$$M_1 = 4$$
$$M_2 = 4$$

$$F_0(z) = (1/\sqrt{2})(-z^{+1} + 1)$$
$$F_1(z) = (1/2)(-z^{+3} - z^{+2} + z^{+1} + 1)$$
$$F_2(z) = (1/2)(z^{+3} + z^{+2} + z^{+1} + 1)$$

(b) Compute $x[n]$ for

$$\beta^{(1)}(z) = \sqrt{2}\left(1 + z^{-1}\right)$$
$$\beta^{(2)}(z) = 0$$
$$\alpha^{(2)}(z) = 2z^{-1}$$

Solution:

$$\begin{aligned}\beta^{(1)}(z)(z^2) &= \sqrt{2}\left(1 + z^{-2}\right)\\ \beta^{(2)}(z)(z^4) &= 0\\ \alpha^{(2)}(z^4) &= 2z^{-4}\end{aligned}$$

$$\begin{aligned}X_0(z) = \beta^{(1)}(z)(z^2)F_0(z) &= -z^{+1} + 1 - z^{-1} + z^{-2}\\ X_1(z) = \beta^{(2)}(z)(z^4)F_1(z) &= 0\\ X_2(z) = \alpha^{(2)}(z^4)F_2(z) &= z^{-1} + z^{-2} + z^{-3} + z^{-4}\end{aligned}$$

$$\begin{aligned}X(z) &= -z^{+1} + 1 - z^{-1} + z^{-2} + z^{-1} + z^{-2} + z^{-3} + z^{-4}\\ &= -z^{+1} + 1 + 2z^{-2} + z^{-3} + z^{-4}\end{aligned}$$

Hence, we get the output

$$x[n] = -\delta[n+1] + \delta[n] + 2\delta[n-2] + \delta[n-3] + \delta[n-4]$$