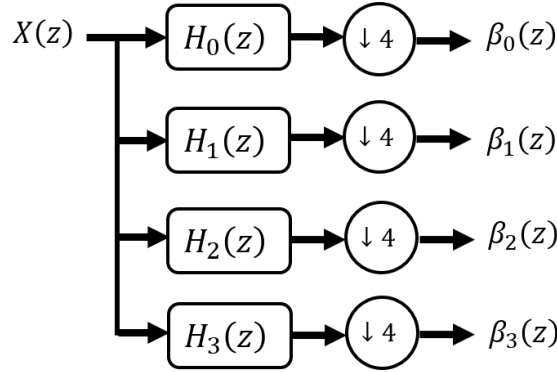**Question #1:**   Consider the four-channel analysis filter bank below.

Let the four filters and three inputs be defined by

$$H_0(z) = (1/2)\left[z^{+3} + z^{+2} + z^{+1} + 1\right]$$
$$H_1(z) = (1/2)\left[-z^{+3} - z^{+2} + z^{+1} + 1\right]$$
$$H_2(z) = (1/2)\left[z^{+3} - z^{+2} - z^{+1} + 1\right]$$
$$H_3(z) = (1/2)\left[z^{+3} - z^{+2} + z^{+1} - 1\right]$$

Let the input be defined by

$$X(z) = 2z^{-4} + 2z^{-7}$$

Compute $\beta_0(z)$, $\beta_1(z)$, $\beta_2(z)$, and $\beta_3(z)$.

Solution: The filtered signals are:

$$
\begin{aligned}
X(z)H_0(z) &= (1/2)\left[z^{+3} + z^{+2} + z^{+1} + 1\right]\left[2z^{-4} + 2z^{-7}\right] \\
&= z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-4} + z^{-5} + z^{-6} + z^{-7} \\
&= z^{-1} + z^{-2} + z^{-3} + 2z^{-4} + z^{-5} + z^{-6} + z^{-7} \\
X(z)H_1(z) &= (1/2)\left[-z^{+3} - z^{+2} + z^{+1} + 1\right]\left[2z^{-4} + 2z^{-7}\right] \\
&= -z^{-1} - z^{-2} + z^{-3} + z^{-4} - z^{-4} - z^{-5} + z^{-6} + z^{-7} \\
&= -z^{-1} - z^{-2} + z^{-3} - z^{-5} + z^{-6} + z^{-7} \\
X(z)H_2(z) &= (1/2)\left[z^{+3} - z^{+2} - z^{+1} + 1\right]\left[2z^{-4} + 2z^{-7}\right] \\
&= z^{-1} - z^{-2} - z^{-3} + z^{-4} + z^{-4} - z^{-5} - z^{-6} + z^{-7} \\
&= z^{-1} - z^{-2} - z^{-3} + 2z^{-4} - z^{-5} - z^{-6} + z^{-7} \\
X(z)H_3(z) &= (1/2)\left[z^{+3} - z^{+2} + z^{+1} - 1\right]\left[2z^{-4} + 2z^{-7}\right] \\
&= z^{-1} - z^{-2} + z^{-3} - z^{-4} + z^{-4} - z^{-5} + z^{-6} - z^{-7} \\
&= z^{-1} - z^{-2} + z^{-3} - z^{-5} + z^{-6} - z^{-7}
\end{aligned}
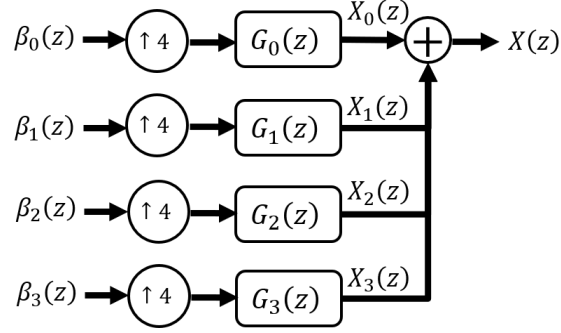$$

The coefficients are:

$$\beta_0(z) = 2z^{-1}$$
$$\beta_1(z) = 0$$
$$\beta_2(z) = 2z^{-1}$$
$$\beta_3(z) = 0$$

Question #2: Consider the four-channel synthesis filter bank below.

Let the four filters and three inputs be defined by

$$G_0(z) = (1/2) \left[1 + z^{-1} + z^{-2} + z^{-3} \right] \qquad \beta_0(z) = z^{-1}$$
$$G_1(z) = (1/2) \left[1 + z^{-1} - z^{-2} - z^{-3} \right] \qquad \beta_1(z) = 1 + 3z^{-1}$$
$$G_2(z) = (1/2) \left[1 - z^{-1} - z^{-2} + z^{-3} \right] \qquad \beta_2(z) = 0$$
$$G_3(z) = (1/2) \left[1 - z^{-1} + z^{-2} - z^{-3} \right] \qquad \beta_3(z) = 0$$

Compute $X_0(z)$, $X_1(z)$, $X_2(z)$, $X_3(z)$, and $X(z)$.

Solution:

$$X_0(z) = (1/2) \left[z^{-4} + z^{-5} + z^{-6} + z^{-7} \right]$$
$$X_1(z) = (1/2) \left[1 + z^{-1} - z^{-2} - z^{-3} + 3z^{-4} + 3z^{-5} - 3z^{-6} - 3z^{-7} \right]$$
$$X_2(z) = 0$$
$$X_3(z) = 0$$
$$X(z) = (1/2) \left[1 + z^{-1} - z^{-2} - z^{-3} + 4z^{-4} + 4z^{-5} - 2z^{-6} - 2z^{-7} \right]$$