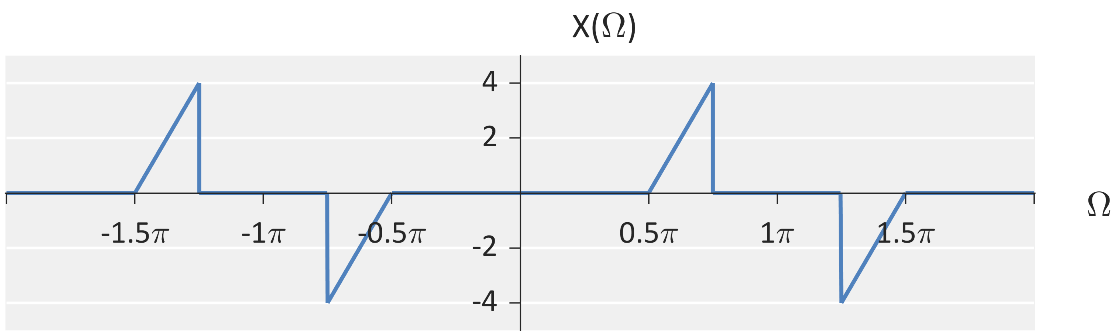
Full Name: ______________________________

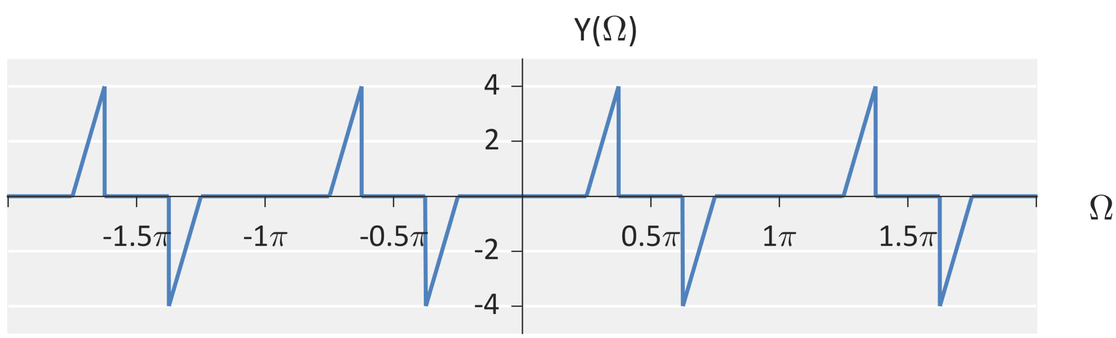
EEE 5502 (Fall 2022) – Practice **Upsampling / Interpolation, 2022**

Question #1: Consider the DTFT of $x[n]$, shown below.



(a) *(3 pts)* Sketch the DTFT of $x[n]$ after upsampling by 2 (without an interpolation filter).

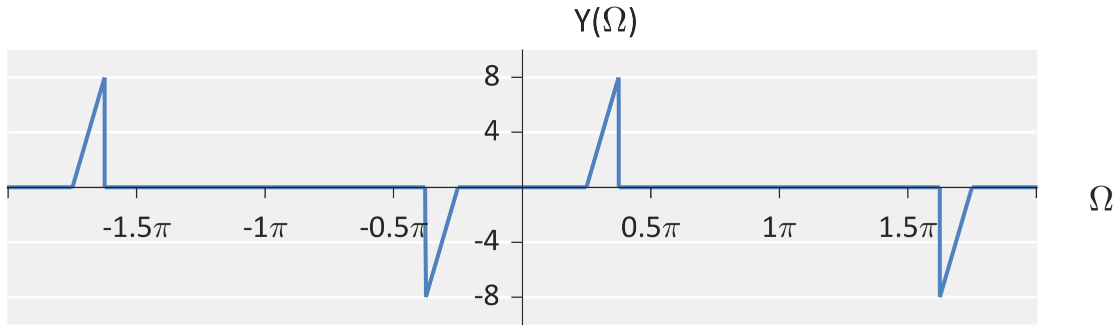
Solution: When we upsample by 2, we shrink the signal by 2. As a result, the value at 0.5π shrinks to 0.25π and the value at 0.75π shrinks to 0.375π. This shrinking process is the same across the entire signal. Upsampling alone does not have an amplitude change. Therefore, our result is shown below.



(b) *(3 pts)* Sketch the DTFT of $x[n]$ after upsampling by 2 (with an interpolation filter).

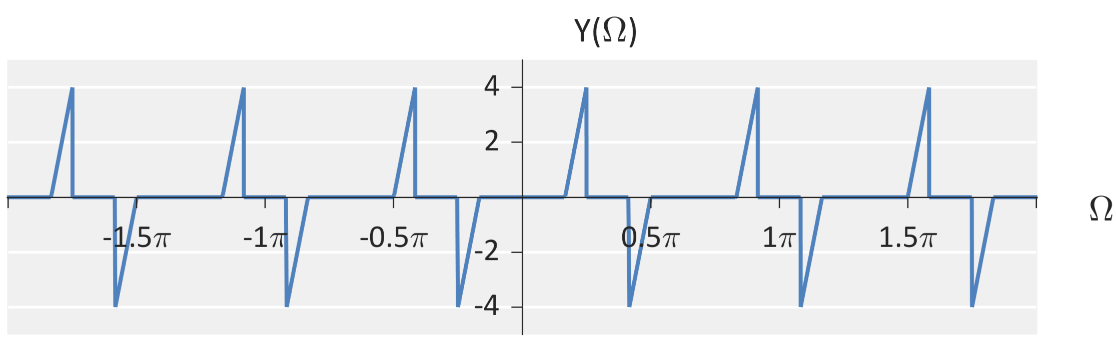
Solution: When we upsample by 2, we shrink the signal by 2. As a result, the value at 0.5π shrinks to 0.25π and the value at 0.75π shrinks to 0.375π. This shrinking process is the same across the entire signal. The result of this is shown in the previous question.

When we include the interpolation filter, we then apply a low-pass filter with a cut-off frequency of π/M and a gain of M, where M is our upsampling factor. Therefore, we only keep the signal between $-\pi/2$ to $\pi/2$ (and in every equivalent 2π period). Our result is shown below.



(c) *(3 pts)* Sketch the DTFT of $x[n]$ after upsampling by 3 (without an interpolation filter).

Solution: When we upsample by 3, we shrink the signal by 3. As a result, the value at 0.5π shrinks to $(1/6)\pi$ and the value at 0.75π shrinks to $(3/12)\pi = (1/4)\pi$. This shrinking process is the same across the entire signal. Upsampling alone does not have an amplitude change. Therefore, our result is shown below.



(d) *(3 pts)* Sketch the DTFT of $x[n]$ after upsampling by 3 (with an interpolation filter).

Solution: When we upsample by 3, we shrink the signal by 3. As a result, the value at 0.5π shrinks to $(1/6)\pi$ and the value at 0.75π shrinks to $(3/12)\pi = (1/4)\pi$. This shrinking process is the same across the entire signal. The result of this is shown in the previous question.

When we include the interpolation filter, we then apply a low-pass filter with a cut-off frequency of π/M and a gain of M, where M is our upsampling factor. Therefore, we only keep the signal between $-\pi/3$ to $\pi/3$ (and in every equivalent 2π period). Our result is shown below.