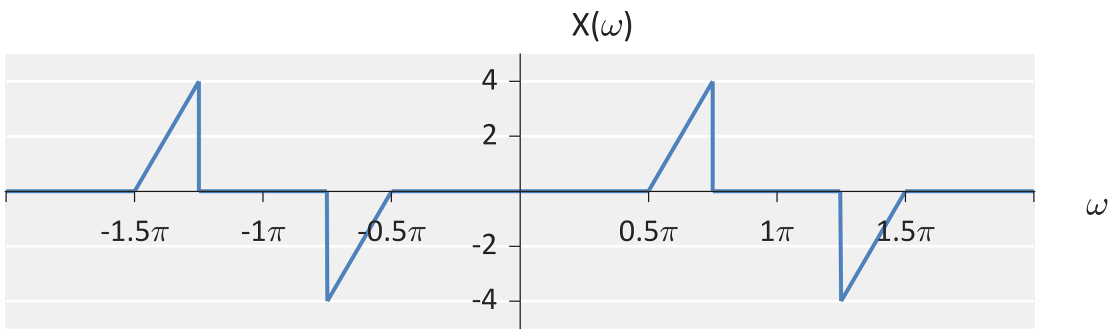
Full Name: ____________________

EEE 5502 (Fall 2022) – Practice **Downsampling / Decimation, 2022**

Question #1: Consider the DTFT of $x[n]$, shown below.

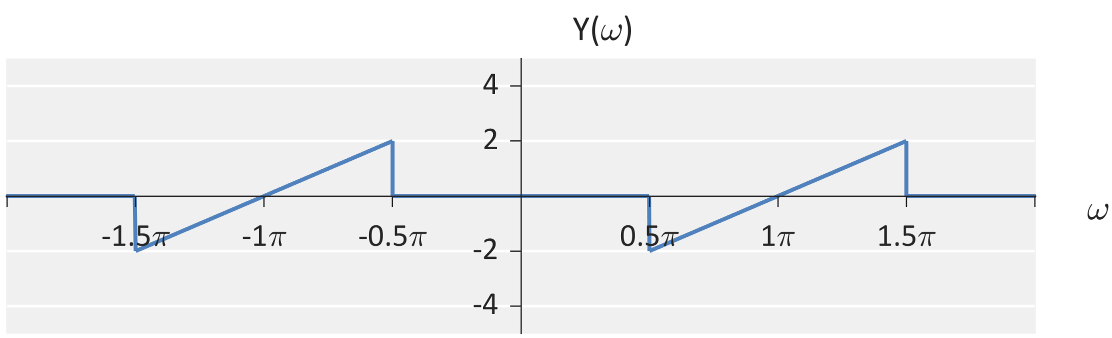


(a) *(4 pts)* Sketch the DTFT of $x[n]$ after downsampling by 2 (without an anti-aliasing filter).

Solution: When we downsample by 2, we stretch the signal from π to $-\pi$ by 2 (with the origin at 0). We also stretch the signal from -3π to $-\pi$ by 2 (with the origin at -2π). We also stretch the signal from π to 3π by 2 (with the origin at 2π). This process repeats every 2π in Ω.

As a result, the value at 0.5π stretches to π and the value at 0.75π stretches to 1.5π. The same happens in the negative Ω range. In addition, the value at 1.5π (0.5π away from 2π) stretches to 1.0π (π away from 2π) and the value at 1.25π (0.75π away from 2π) stretches to 0.5π (1.5π away from 2π). The same happens in the negative Ω range.

Finally, the signal amplitude is multiplied by $1/N$, where N is the downsampling rate, due to downsampling. In this case, $N = 2$. Therefore, our final result is shown below.

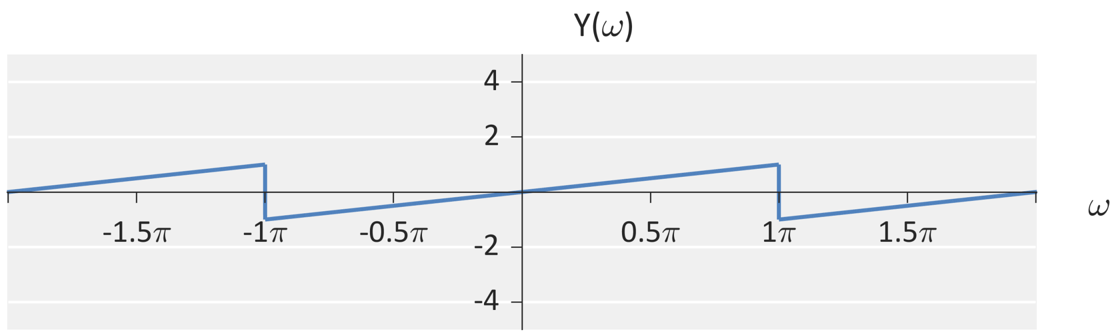


(b) *(4 pts)* Sketch the DTFT of $x[n]$ after downsampling by 4 (without an anti-aliasing filter).

Solution: When we downsample by 4, we stretch the signal from π to $-\pi$ by 4 (with the origin at 0). We also stretch the signal from -3π to $-\pi$ by 4 (with the origin at -2π). We also stretch the signal from π to 3π by 4 (with the origin at 2π). This process repeats every 2π in Ω.

As a result, the value at 0.5π stretches to 2π and the value at 0.75π stretches to 3π. The same happens in the negative Ω range. In addition, the value at 1.5π (0.5π away from 2π) stretches to 0 (2π away from 2π) and the value at 1.25π (0.75π away from 2π) stretches to $-\pi$ (3π away from 2π). The same happens in the negative Ω range and at other increments of 2π.

Finally, the signal amplitude is multiplied by $1/N$, where N is the downsampling rate, due to downsampling. In this case, $N = 4$. Therefore, our final result is shown below.



(c) *(4 pts)* Sketch the DTFT of $x[n]$ after downsampling by 2 (with an anti-aliasing filter).

Solution: When we downsample with anti-aliasing, we first apply a low-pass filter with a cut-off frequency of π/N, where N is our downsampling factor. Therefore, we only keep the original signal between $-\pi/2$ to $\pi/2$ (and in every equivalent 2π period). In this regions, the signal is always zero. Therefore, our result is shown below.