Full Name: ______________________________

EEE 5502 (Fall 2022) – Practice **Filter Transformations, 2022**

Question #1: Consider the following prototype low-pass filter

$$H(z) = \frac{0.5}{1 - 0.5z^{-1}} \ .$$

(a) Transform the filter to be a high-pass filter with the same cut-off frequency. Determine the resulting transfer function.

Solution:

$$H_{high}(z) = \frac{0.5}{1 + 0.5z^{-1}} \ .$$

(b) Transform the filter to be a bandpass-pass filter with the same overall width and a center frequency of $\pi/2$. Determine the resulting transfer function. Ensure the impulse response is real-valued.

Solution:

$$\begin{aligned} H_{band}(z) &= H_{low}(ze^{j\pi/2}) + H_{low}(ze^{j\pi/2}) \\ &= H_{low}(jz) + H_{low}(-jz) \\ &= \frac{0.5}{1 - 0.5(jz)^{-1}} + \frac{0.5}{1 - 0.5(-jz)^{-1}} \\ &= \frac{0.5}{1 + 0.5jz^{-1}} + \frac{0.5}{1 - 0.5jz^{-1}} \end{aligned}$$

(c) Identify the cut-off frequency ω_c for this filter, defined as the frequency in which

$$|H(\omega)|^2 = (1/2)H(1)$$

Solution:

$$
\begin{aligned}
|H(z)|^2 = \frac{0.25}{|1 - 0.5e^{-j\omega}|^2} &= \frac{1}{2} \\
\frac{0.25}{1 - 0.5e^{-j\omega} - 0.5e^{+j\omega} + 0.25} &= \frac{1}{2} \\
\frac{0.25}{1 - \cos(\omega) + 0.25} &= \frac{1}{2} \\
\frac{0.25}{1.25 - \cos(\omega)} &= \frac{1}{2} \\
\frac{0.25}{1.25 - \cos(\omega)} &= \frac{1}{2} \\
\frac{1}{5 - 4\cos(\omega)} &= \frac{1}{2} \\
5 - 4\cos(\omega) &= 2 \\
-4\cos(\omega) &= -3 \\
\cos(\omega) &= 0.75 \\
\omega &= \cos^{-1}(0.75) \approx 0.723
\end{aligned}
$$