

Question #1: Consider the following prototype low-pass filter

$$H(z) = \frac{0.5}{1 - 0.5z^{-1}} .$$

(a) Transform the filter to be a high-pass filter with the same cut-off frequency. Determine the resulting transfer function.

(b) Transform the filter to be a bandpass-pass filter with the same overall width and a center frequency of $\pi/2$. Determine the resulting transfer function. Ensure the impulse response is real-valued.

(c) Identify the cut-off frequency ω_c for this filter, defined as the frequency in which

$$|H(\omega)|^2 = (1/2)H(1)$$