Full Name: ______________________________

EEE 5502 (Fall 2022) – Practice **Designing IIR Filters, 2022**

Question #1: Consider a continuous-time filter defined by the transfer function

$$H(s) = \frac{s-2}{s^2-1} = \frac{(3/2)}{s+1} - \frac{(1/2)}{s-1}$$

(a) Compute $H(z)$, the z-transform of discrete-time filter using the discrete-time approximation for the difference equation. Assume a sampling period of $T_s = 1$. Determine the poles and zeros.

Solution: Let

$$s \to \frac{1}{T_s}(1 - z^{-1})$$

So

$$\begin{aligned} H(z) &= \frac{1 - z^{-1} - 2}{(1-z^{-1})^2 - 1} \\ &= \frac{-z^{-1} - 1}{(1-z^{-1})^2 - 1} \\ &= \frac{-z - z^2}{(z-1)^2 - z^2} \\ &= \frac{-z - z^2}{z^2 - 2z + 1 - z^2} \\ &= \frac{-z - z^2}{-2z + 1} \\ &= \frac{z^2 + z}{2(z - 1/2)} \\ &= \frac{z(z+1)}{2(z-1/2)} \end{aligned}$$

The zeros are at $z = 0, -1$ and the poles are at $z = 1/2$.

(b) Compute $G(z)$, the z-transform of discrete-time filter using the impulse invariance method. Assume a sampling period of $T_s = 1$.

Solution: The poles of $H(s)$ are $s = 1, -1$. So the poles of the $H(z)$ will be $z = e^{+1}, e^{-1}$.

$$G(z) = \frac{(3/2)}{1 - e^{-1}z^{-1}} - \frac{(1/2)}{1 - e^{+1}z^{-1}}$$

(c) Compute $R(z)$, the z-transform of discrete-time filter using the bilinear transform. Assume a sampling period of $T_s = 1$. Determine the poles and zeros.

Solution: Let

$$s \rightarrow \frac{2}{T_s} \frac{1 - z^{-1}}{1 + z^{-1}}$$

So

$$\begin{aligned}
H(z) &= \frac{2\frac{1-z^{-1}}{1+z^{-1}} - 2}{\left(2\frac{1-z^{-1}}{1+z^{-1}}\right)^2 - 1} \\
&= \frac{2(1 - z^{-1})(1 + z^{-1}) - 2(1 + z^{-1})^2}{4\,(1 - z^{-1})^2 - (1 + z^{-1})^2} \\
&= \frac{2(1 - z^{-2}) - 2(1 + 2z^{-1} + z^{-2})}{4(1 - 2z^{-1} + z^{-2}) - (1 + 2z^{-1} + z^{-2})} \\
&= \frac{-4(z^{-2} + z^{-1})}{(3 - 10z^{-1} + 3z^{-2})} \\
&= \frac{-4(z^{-2} + z^{-1})}{3(1 - (10/3)z^{-1} + z^{-2})} \\
&= \frac{-4}{3} \frac{z + 1}{z^2 - (10/3)z + 1}
\end{aligned}$$

The zeros are at $z = -1$. The poles are at $z = 1/3, 3$.