Question #1: Answer the following questions.

(a) Prove that the frequency response $|H(\omega)|e^{-j\omega\tau}$ always represents a linear phase filter.

Solution: $|H(\omega)|$ has zero phase (making it linear phase) and is therefore purely real. As previously proved, a purely frequency response has even symmetry in time (also making it linear phase). The $e^{-j\omega\tau}$ is simply a delay in time / adds a linear phase in frequency.

(b) (True or False) An IIR filter cannot be a stable and linear phase filter. Justify.

Solution: False. An impulse response like $h[n] = (0.4)^{|n|}$ is stable, linear phase, and IIR.

(c) (True or False) An IIR filter cannot be causal and linear phase simultaneously. Justify.

Solution: True. In the pole-zero plot, a linear phase filter has poles or zeros mirrored across the unit circle. If poles are mirrored across the unit circle and the system is causal, then the region of convergence will always be outside the unit circle.

Question #2: Consider a desired magnitude response defined by

$$H_d(\omega) = \sum_{k=-\infty}^{\infty} u(\omega + \pi/2 - 2\pi k) - u(\omega - \pi/2 - 2\pi k)$$

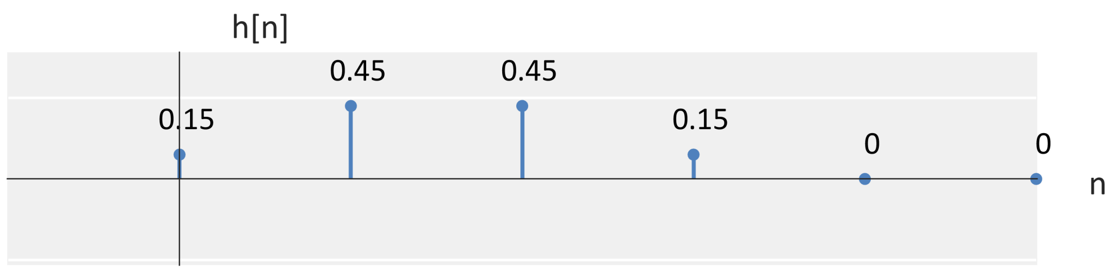
(a) Design discrete-time FIR filter coefficients $h[n]$ that approximate a filter with magnitude response $|H_d(\omega)|$ with the windowing method of length $N = 4$. Force the impulse response to be causal and linear-phase (thru shifting if necessary). Sketch $h[n]$.

Solution:

$$h_d[n] = \frac{1}{2}\text{sinc}((\pi/2)n)$$

The windowed impulse response needs to be shifted by 1.5 samples and is

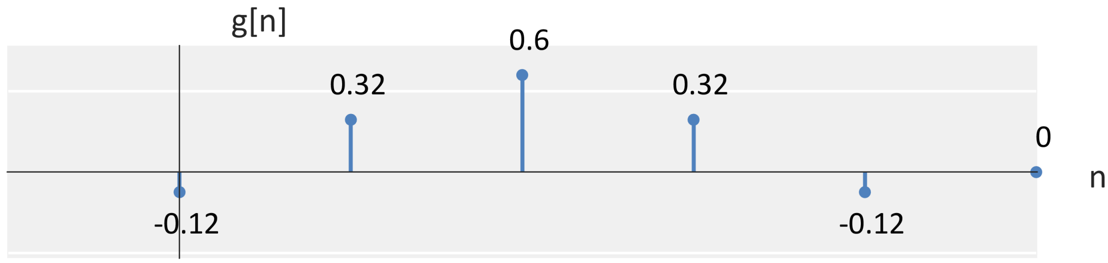
$$\begin{aligned} h[n] &= \frac{1}{2}\,\text{sinc}((\pi/2)(n-3/2))w[n] \\ &= \frac{1}{2}\,\text{sinc}((\pi/2)(n-3/2))\,[u[n]-u[n-4]] \end{aligned}$$



(b) Design discrete-time FIR filter coefficients $g[n]$ that approximate a filter with frequency response $|H_d(\omega)|$ with a 5-point frequency sampling method.

Solution: The sampled frequency response $H[k]$ from frequencies $\omega_k = 2\pi(k/5)$. So our frequencies are $\omega_k = 0, 2\pi/5, 4\pi/5, 6\pi/5, 8\pi/5$. The frequencies on one-half of the unit circle are $\omega_k = 0, 2\pi/5, 4\pi/5$. So our impulse rsponse is

$$\begin{aligned} g[n] &= \frac{1}{5}\left[1 + 2(-1)(1)\cos\left(\frac{2\pi}{5}\left(n+\frac{1}{2}\right)\right)\right][u[n]-u[n-5]] \\ &= \frac{1}{5}\left[1 - 2\cos\left(\frac{2\pi}{5}\left(n+\frac{1}{2}\right)\right)\right][u[n]-u[n-5]] \end{aligned}$$


g[n]
0.6
0.32
0.32
0
n
-0.12
-0.12