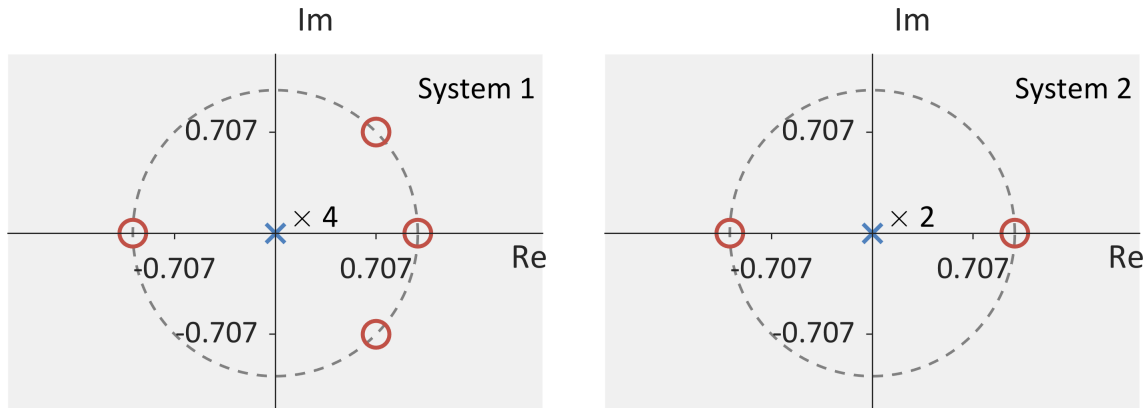


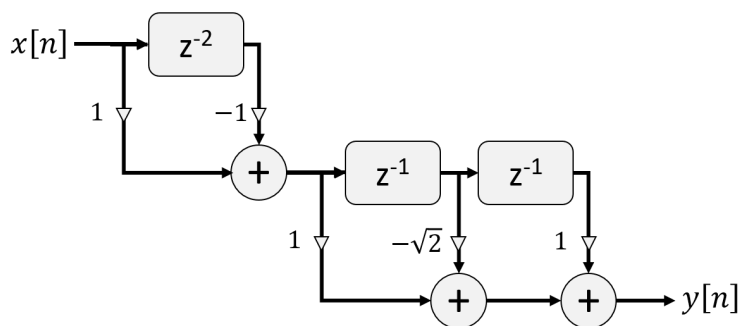
Question #1: Consider the systems given by the following pole-zero plots.



- (a) Draw a block diagram of the FIR cascade form implementation for System 1. Design the implementation so that complex conjugate pairs are together. **Ensure that you use real values on the connections in the block diagram.**

Solution:

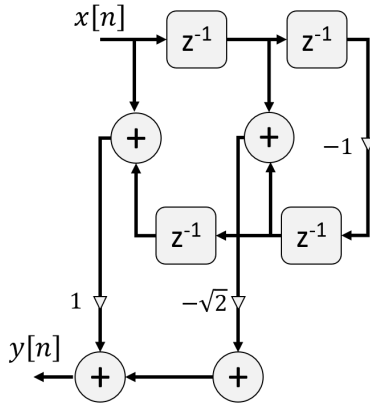
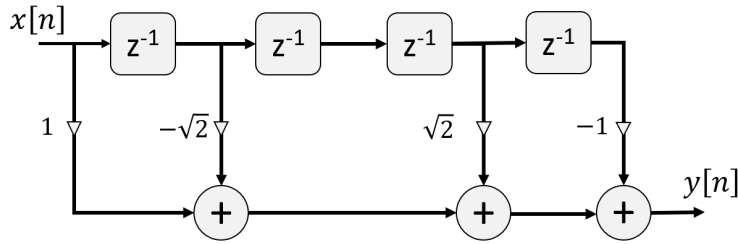
$$\begin{aligned}
 H(z) &= (1 - z^{-1})(1 + z^{-1})(1 - z^{-1}e^{j\pi/4})(1 - z^{-1}e^{-j\pi/4}) \\
 &= (1 - z^{-2})(1 - 2\cos(\pi/4)z^{-1} + z^{-2}) \\
 &= (1 - z^{-2})(1 - \sqrt{2}z^{-1} + z^{-2})
 \end{aligned}$$



- (b) Draw a block diagram of the FIR direct form implementation for System 1. Design the implementation so that complex conjugate pairs are together. **Ensure that you use real values on the connections in the block diagram.**

Solution:

$$\begin{aligned}
 H(z) &= (1 - z^{-1}) (1 + z^{-1}) (1 - z^{-1}e^{j\pi/4}) (1 - z^{-1}e^{-j\pi/4}) \\
 &= (1 - z^{-1}) (1 + z^{-1}) (1 - 2\cos(\pi/4)z^{-1} + z^{-2}) \\
 &= (1 - z^{-2}) (1 - 2\cos(\pi/4)z^{-1} + z^{-2}) \\
 &= (1 - 2\cos(\pi/4)z^{-1} + z^{-2}) - (z^{-2} - 2\cos(\pi/4)z^{-3} + z^{-4}) \\
 &= 1 - 2\cos(\pi/4)z^{-1} + 2\cos(\pi/4)z^{-3} - z^{-4} \\
 &= 1 - \sqrt{2}z^{-1} + \sqrt{2}z^{-3} - z^{-4}
 \end{aligned}$$



- (c) Draw a block diagram of the FIR frequency sampling implementation for System 2. **You may use complex values for the connections in the block diagram.**

Solution:

$$\begin{aligned}
 H(z) &= (1 - z^{-1}) (1 + z^{-1}) \\
 &= (1 - z^{-2})
 \end{aligned}$$

The DFT of the impulse response is defined by

$$\begin{aligned}
 H[k] &= \sum_{n=0}^2 x[n] e^{-j(2\pi k/3)n} \\
 &= (1)e^{-j(2\pi k/3)(0)} + (0)e^{-j(2\pi k/3)(1)} + (-1)e^{-j(2\pi k/3)(2)} \\
 &= (1)e^{-j(2\pi k/3)(0)} + (-1)e^{-j(2\pi k/3)(2)} \\
 &= 1 - e^{-j(4\pi k/3)} \\
 &= [1.500 + j(0.8660)] \delta[k - 1] + [1.500 - j(0.8660)] \delta[k - 2] \\
 &= \sqrt{3}e^{-j\pi/6} \delta[k - 1] + \sqrt{3}e^{j\pi/6} \delta[k - 2]
 \end{aligned}$$

