Full Name: ______________________________

EEE 5502 (Fall 2022) – Practice **Discrete Fourier Transform, 2022**

Question #1: Consider the length-4 signal $x[n]$ with values

$$\{1 \quad 1 \quad 0 \quad 0\}$$

(a) Compute the length-4 discrete Fourier transform (DFT) of $x[n]$ to get $X[k]$.

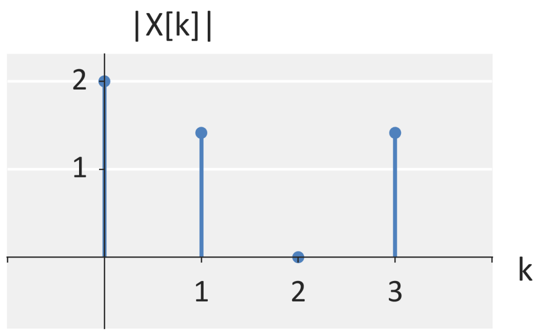
Solution:

$$\begin{aligned} X[k] &= \sum_{n=0}^{N-1} x[n]e^{-j(2\pi k/N)n} \\ &= e^{-j(2\pi k/N)0} + e^{-j(2\pi k/N)(1)} \\ &= 1 + e^{-j(2\pi k/N)} \\ &= 1 + e^{-j(\pi k/2)} \end{aligned}$$

(b) Sketch the length-4 magnitude of the DFT $|X[k]|$.

Solution:

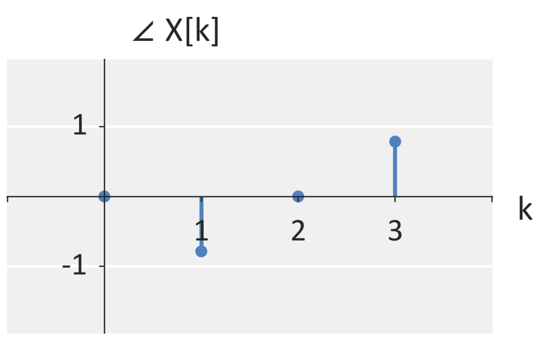
$$\begin{aligned} X[k] &= 1 + e^{-(j\pi k/2)} \\ X[0] &= 2 \\ X[1] &= 1 - j \\ X[2] &= 0 \\ X[3] &= 1 + j \\ |X[0]| &= 2 \\ |X[1]| &= \sqrt{2} \\ |X[2]| &= 0 \\ |X[3]| &= \sqrt{2} \end{aligned}$$



(c) Sketch the length-4 phase of the DFT $\angle X[k]$.

Solution:

$$\begin{aligned}\angle X[0] &= 0\\ \angle X[1] &= -\pi/4\\ \angle X[2] &= 0\\ \angle X[3] &= \pi/4\end{aligned}$$



Question #2: Consider the length-4 signal $y[n]$ with values

$$\{1 \quad 0 \quad 0 \quad 1\}$$

(a) Compute the length-4 discrete Fourier transform (DFT) of $y[n]$ to get $Y[k]$.

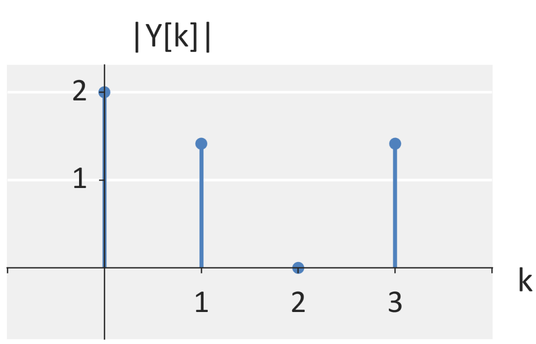
Solution:

$$\begin{aligned} Y[k] &= \sum_{n=0}^{N-1} y[n] e^{-j(2\pi k/N)n} \\ &= e^{-j(2\pi k/N)0} + e^{-j(2\pi k/N)(3)} \\ &= 1 + e^{-j(2\pi k/N)3} \\ &= 1 + e^{-j(3\pi k/2)} \end{aligned}$$

(b) Sketch the length-4 magnitude of the DFT $|Y[k]|$.

Solution:

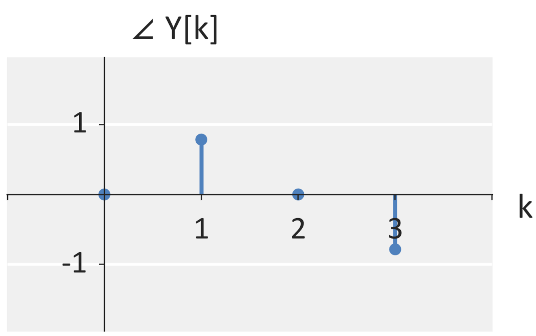
$$\begin{aligned} Y[k] &= 1 + e^{-(j3\pi k/2)} \\ Y[0] &= 2 \\ Y[1] &= 1 + j \\ Y[2] &= 0 \\ Y[3] &= 1 - j \\ |Y[0]| &= 2 \\ |Y[1]| &= \sqrt{2} \\ |Y[2]| &= 0 \\ |Y[3]| &= \sqrt{2} \end{aligned}$$



(c) Sketch the length-4 phase of the DFT $\angle Y[k]$.

Solution:

$$\begin{aligned}\angle Y[0] &= 0\\ \angle Y[1] &= \pi/4\\ \angle Y[2] &= 0\\ \angle Y[3] &= -\pi/4\end{aligned}$$



(d) Explain the similarities and differences between $|X[k]|$ from the previous problem, $|Y[k]|$, $\angle X[k]$ from the previous problem, and $\angle Y[k]$.

Solution: The magnitudes are the same but the phases are different. This is due to the assumed periodicity of the time-domain signal.