

Question #1: The Nyquist sampling rate of a signal is the minimum rate $\omega_s = 2\pi/T_s$ for which the signal may be sampled for which no aliasing occurs from its samples.

Consider three signals $x_1(t)$ and $x_2(t)$ and $x_3(t)$ such that their Fourier transforms satisfy

$$\begin{aligned}X_1(\Omega) &= 0, & 30 \leq |\Omega| \\X_2(\Omega) &= 0, & |\Omega| \leq 15, |\Omega| \geq 20 \\X_3(\Omega) &= 10^{-\Omega}, & |\Omega| < \infty\end{aligned}$$

Determine the minimum frequency Ω_s at which we must sample the following signals in order to prevent aliasing. If aliasing is unavoidable, write that instead of a frequency.

(a) $x(t) = x_1(t) + x_2(t)$

Solution: $\Omega_s = 2\Omega_{\max} = 60$

(b) $x(t) = x_1(t) * x_3(t)$

Solution: $\Omega_s = 2\Omega_{\max} = 60$

(c) $x(t) = x_1(t)x_2(t)$

Solution: $\Omega_s = 2\Omega_{\max} = 100$

(d) $x(t) = \cos(3.6\pi t + 9.23)$

Solution: $\Omega_s = 2\Omega_{\max} = 7.2\pi$

(e) $x(t) = u(t - 1) - u(t - 4)$

Solution: $\Omega_s = 2\Omega_{\max} = \infty$

(f) $x(t) = \delta(t)$

Solution: $\Omega_s = 2\Omega_{\max} = \infty$