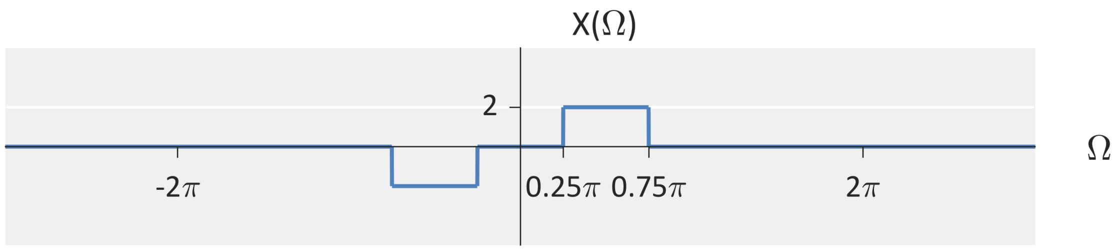
Full Name: ______________________________

EEE 5502 (Fall 2022) – Practice **Sampling, 2022**

Question #1: Consider the Fourier transform of $x(t)$, shown below.



(a) Determine the Nyquist sampling rate for $x(t)$ (in angular frequency).

Solution: The Nyquist sampling rate Ω_N is the lowest rate that we can sample data while retaining of the information in the signal. It is defined by

$$\Omega_N = 2\Omega_{max}$$

The maximum angular frequency Ω_{max} is the largest frequency that has an associated [y-]value of greater than zero. In the plot,

$$\Omega_{max} = 0.75\pi = 3\pi/4\ .$$

Therefore, the Nyquist sampling rate is

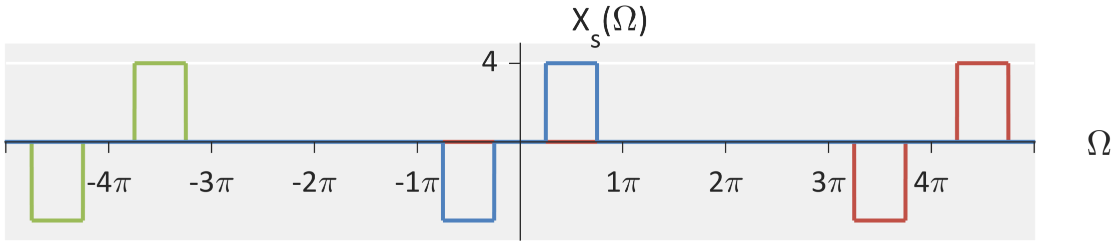
$$\Omega_N = 3\pi/2\ .$$

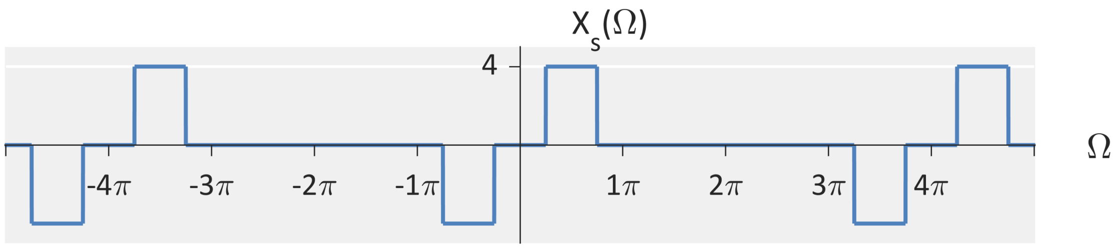
(b) Sketch the Fourier transform $X_s(\Omega)$ of the sampled $X(\Omega)$ with a sampling rate of $\Omega_s = 4\pi$. Do we experience aliasing?

Solution: When we sample our signal, we create copies of our original signal $X(\Omega)$, shift each copy by integer multiples of the sampling rate Ω_s, and multiply the amplitudes by $1/T_s$. We then add all of the shifted copies together.

All of the copies (separated by a period of $\Omega_s = 4\pi$ and multiplied by $1/T_s = \Omega_s/2\pi = 2$) looks like (each shade represents a different signal) the first figure below. Then when we add all of these copies together, we get the second figure below.

Aliasing occurs when the sampling rate is less than the Nyquist rate ($\Omega_s < \Omega_N$). This is not true, so **no aliasing occurs**.

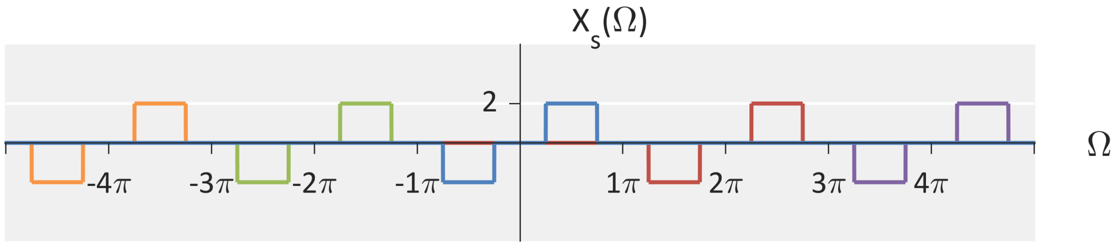


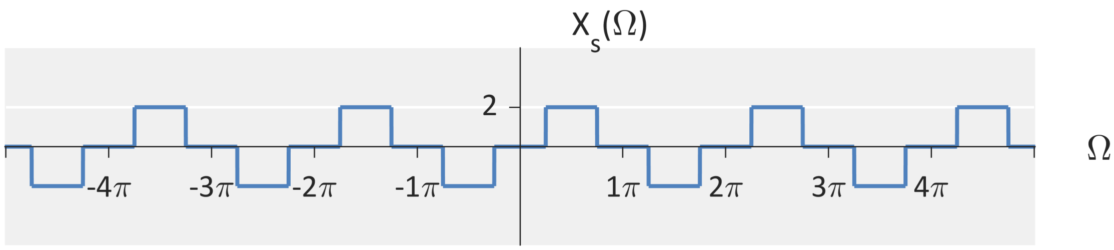


(c) Sketch the Fourier transform $X_s(\Omega)$ of the sampled $X(\Omega)$ with a sampling rate of $\Omega_s = 2\pi$. Do we experience aliasing?

Solution: All of the copies (separated by a period of $\Omega_s = 2\pi$ and multiplied by $1/T_s = \Omega_s/2\pi = 1$) looks like (each shade represents a different signal) the first figure below. Then when we add all of these copies together, we get the second figure below.

Aliasing occurs when the sampling rate is less than the Nyquist rate ($\Omega_s < \Omega_N$). This is not true, so **no aliasing occurs**.



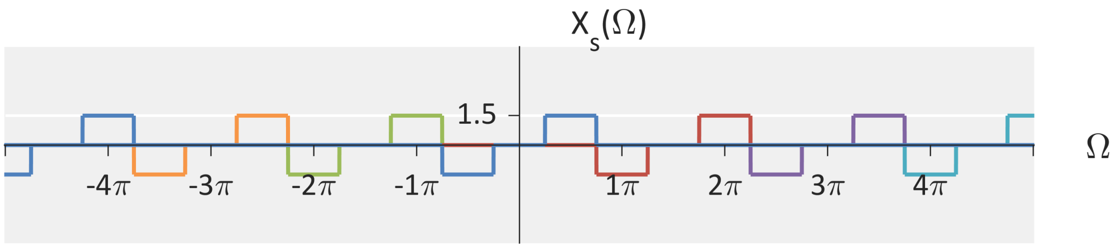


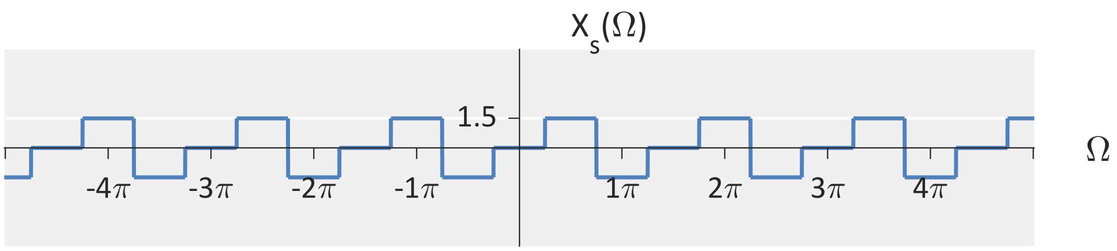
(d) Sketch the Fourier transform $X_s(\Omega)$ of the sampled $X(\Omega)$ with a sampling rate of $\Omega_s = 3\pi/2$.

Do we experience aliasing?

Solution: All of the copies (separated by a period of $\Omega_s = 3\pi/2$ and multiplied by $1/T_s = \Omega_s/2\pi = 3/4$) looks like (each shade represents a different signal) the first figure below. Then when we add all of these copies together, we get the second figure below.

Aliasing occurs when the sampling rate is less than the Nyquist rate ($\Omega_s < \Omega_N$). Since $3\pi/2 = 3\pi/2$, the answer to this question is somewhat ambiguous. If Ω_N contains a value, then aliasing occurs (albeit very little). If Ω_N does not contain a value (instead the values are infinitesimally close to it), then no aliasing occurs. I would accept either answer in an exam / quiz.

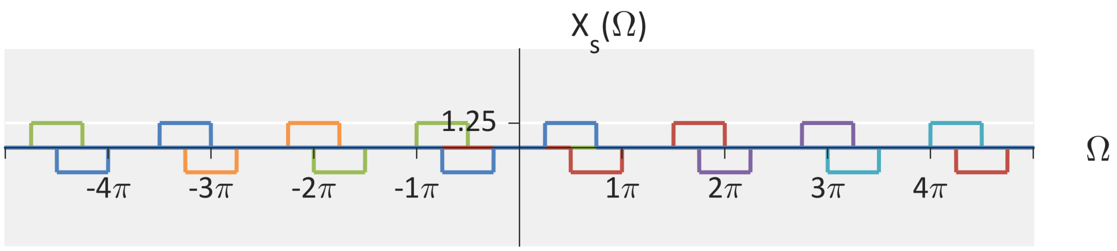


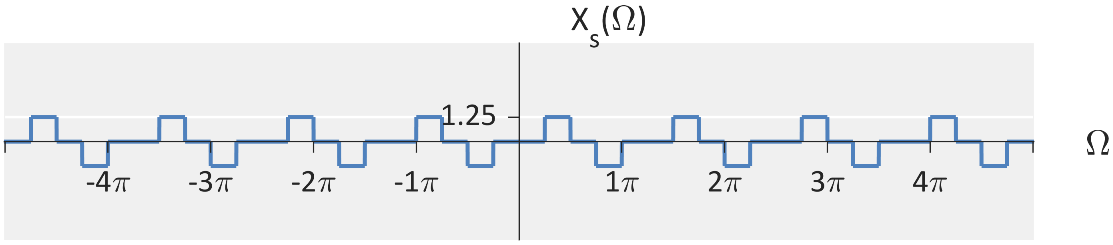


(e) Sketch the Fourier transform $X_s(\Omega)$ of the sampled $X(\Omega)$ with a sampling rate of $\Omega_s = 5\pi/4$. Do we experience aliasing?

Solution: All of the copies (separated by a period of $\Omega_s = 5\pi/4$ and multiplied by $1/T_s = \Omega_s/2\pi = 5/8$) looks like (each shade represents a different signal) the first figure below. Then when we add all of these copies together, we get the second figure below. Since positive and negative values of each copy partially overlap, some of the values sum to 0.

Aliasing occurs when the sampling rate is less than the Nyquist rate ($\Omega_s < \Omega_N$). Since $5\pi/4 < 3\pi/2$, **aliasing occurs**.

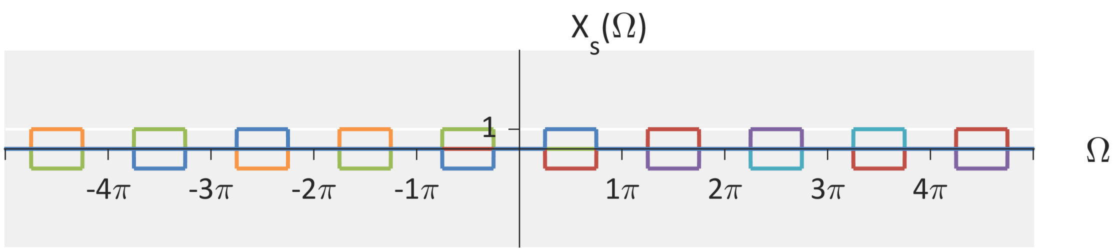




(f) Sketch the Fourier transform $X_s(\Omega)$ of the sampled $X(\Omega)$ with a sampling rate of $\Omega_s = \pi$. Do we experience aliasing?

Solution: All of the copies (separated by a period of $\Omega_s = \pi$ and multiplied by $1/T_s = \Omega_s/2\pi = 1/2$) looks like (each shade represents a different signal) the first figure below. Then when we add all of these copies together, we get the second figure below. Since positive and negative values of each copy overlap, the sum is zero.

Aliasing occurs when the sampling rate is less than the Nyquist rate ($\Omega_s < \Omega_N$). Since $\pi < 3\pi/2$, **aliasing occurs**.

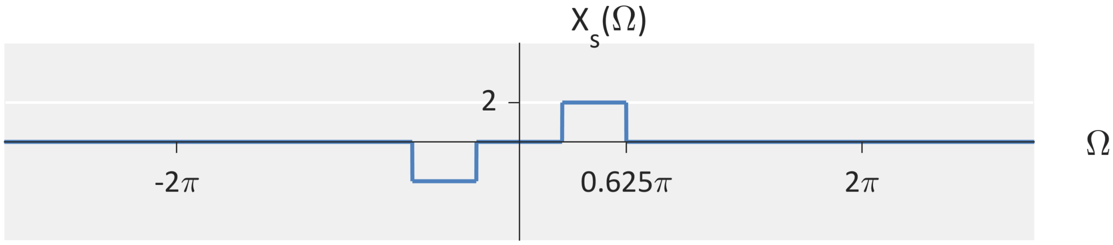


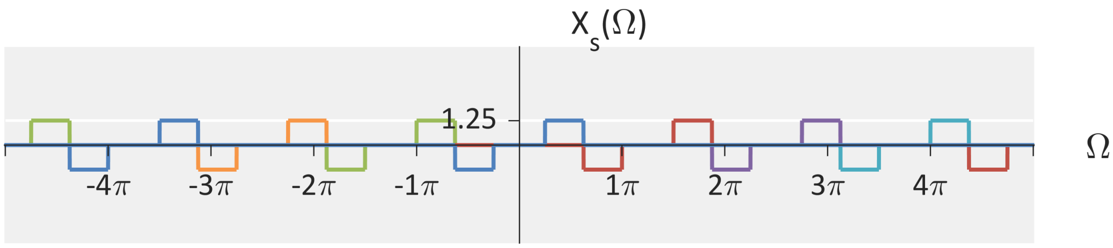
(g) Sketch the Fourier transform $X_s(\Omega)$ of the sampled $X(\Omega)$ with a sampling rate of $\Omega_s = 5\pi/4$ after applying an anti-aliasing filter with cut-off $\Omega_s/2$.

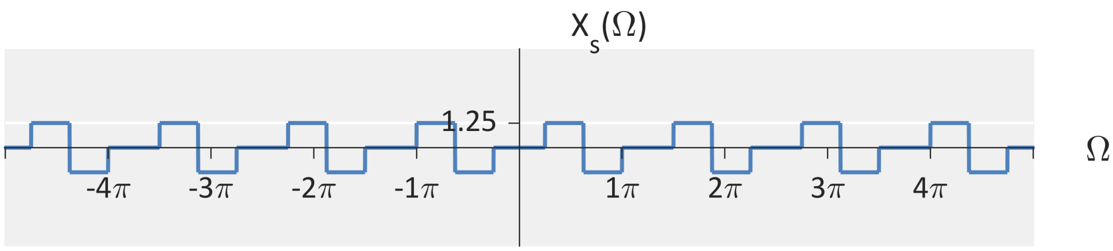
Solution: The anti-aliasing filter is applied before sampling. As a result, the anti-aliased signal that we now sample is the first figure below.

All of the copies (separated by a period of $\Omega_s = 5\pi/4$ and multiplied by $1/T_s = \Omega_s/2\pi = 5/8$) of the anti-aliased signal looks like (each shade represents a different signal) the second figure below. Then when we add all of these copies together, we get the third figure below.

No aliasing occurs due to the anti-aliasing filter.





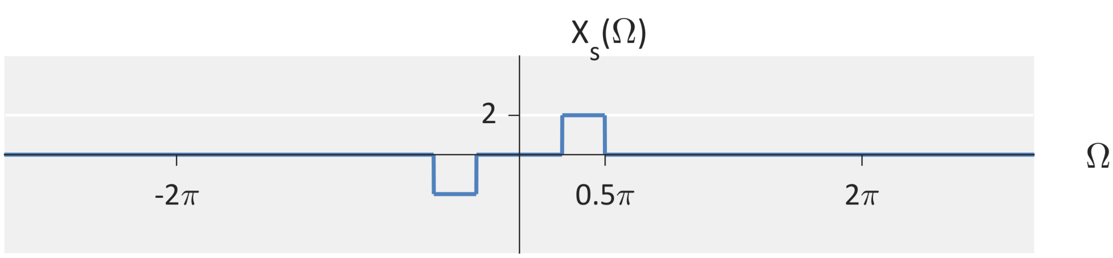


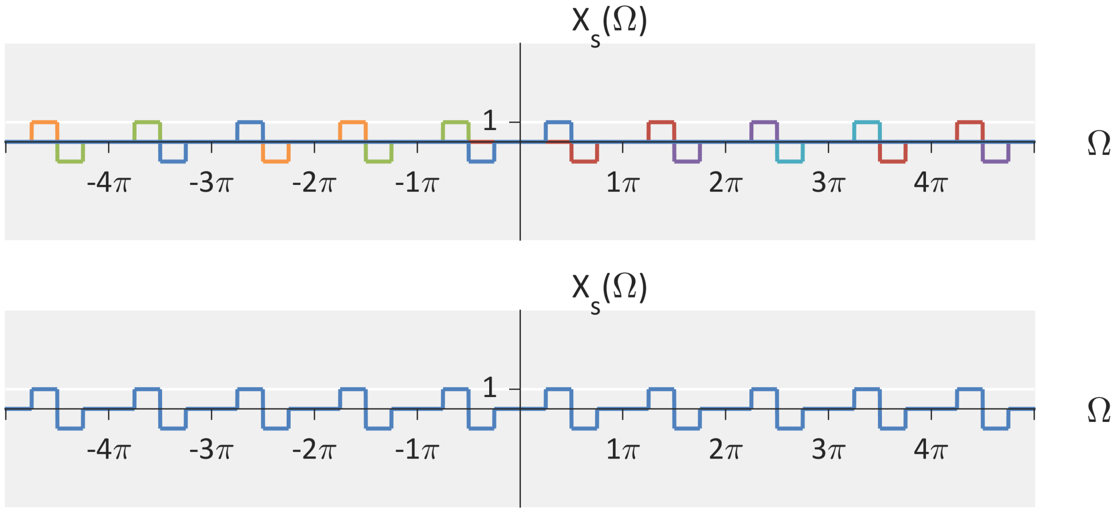
(h) Sketch the Fourier transform $X_s(\Omega)$ of the sampled $X(\Omega)$ with a sampling rate of $\Omega_s = \pi$ after applying an anti-aliasing filter with cut-off $\Omega_s/2$.

Solution: The anti-aliasing is applied before sampling. As a result, the anti-aliased signal that we now sample is the first figure below.

All of the copies (separated by a period of $\Omega_s = \pi$ and multiplied by $1/T_s = \Omega_s/2\pi = 1/2$) of the anti-aliased signal looks like (each shade represents a different signal) the second figure below. Then when we add all of these copies together, we get the third figure below.

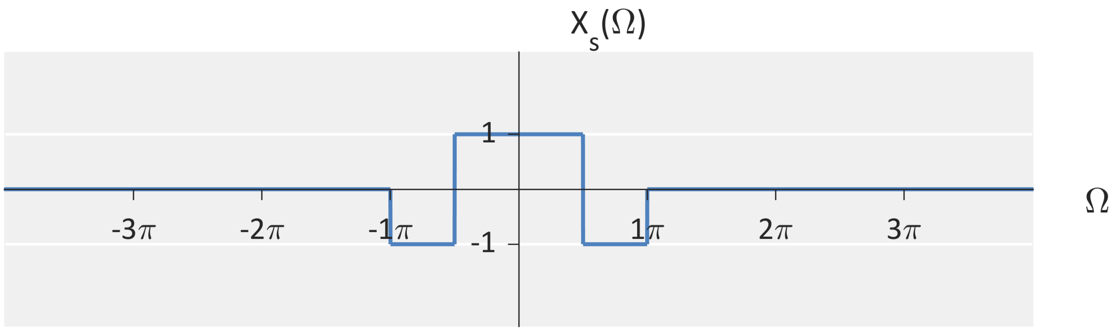
No aliasing occurs due to the anti-aliasing filter.





Xs(Ω)
1
Ω
-4π
-3π
-2π
-1π
1π
2π
3π
4π
Xs(Ω)
1
Ω
-4π
-3π
-2π
-1π
1π
2π
3π
4π

Question #2: Consider the Fourier transform of $x(t)$, shown below



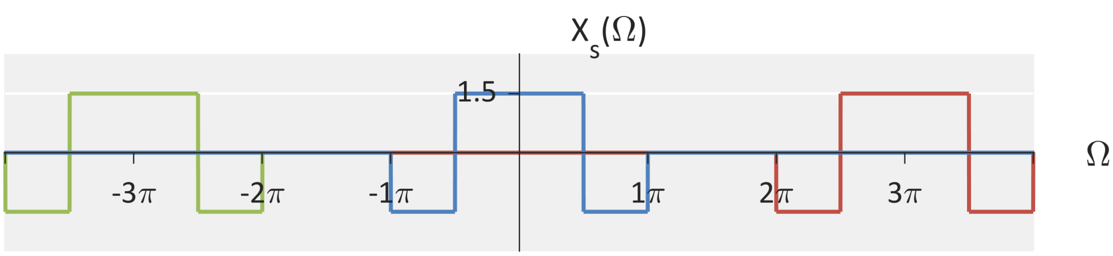
(a) Determine the Nyquist sampling rate for $x(t)$ (in angular frequency).

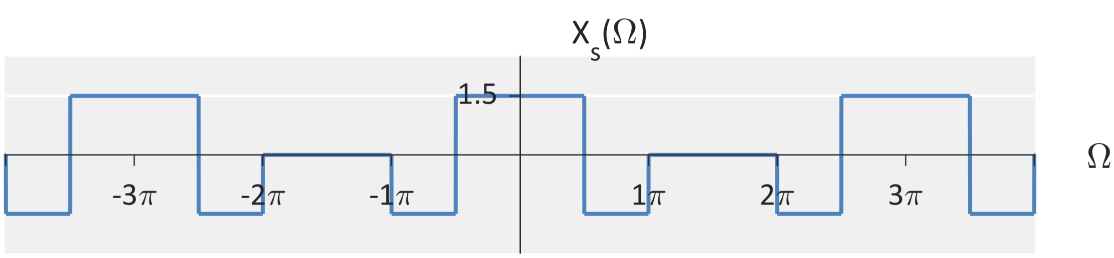
Solution: The Nyquist sampling rate is two times the maximum sampling rate. Hence, the Nyquist sampling rate is $\Omega_s = 2\pi$.

(b) Sketch the Fourier Transform (for $\Omega = -4\pi$ to $\Omega = 4\pi$) of the sampled signal with a sampling rate $\Omega_s = 3\pi$.

Solution: When we sample our signal, we create copies of our original signal $X(\Omega)$, shift each copy by integer multiples of the sampling rate Ω_s, and multiply the amplitudes by $1/T_s$. We then add all of the shifted copies together.

All of the copies (separated by a period of $\Omega_s = 3\pi$ and multiplied by $1/T_s = \Omega_s/2\pi = 3/2$) looks like (each shade represents a different signal) the first figure below. Then when we add all of these copies together, we get the second figure below.





(c) Sketch the Fourier Transform (from $\Omega = -4\pi$ to $\Omega = 4\pi$) of the sampled signal with a sampling rate $\Omega_s = \pi$ **after** applying a low-pass anti-aliasing filter with cutoff-off at $\Omega_s/2$.

Solution: The anti-aliasing filter is applied before sampling. As a result, the anti-aliased signal that we now sample is the first figure below.

All of the copies (separated by a period of $\Omega_s = \pi$ and multiplied by $1/T_s = \Omega_s/2\pi = 1/2$) looks like (each shade represents a different signal) the second figure below. Then when we add all of these copies together, we get the third figure below.