Full Name: _____

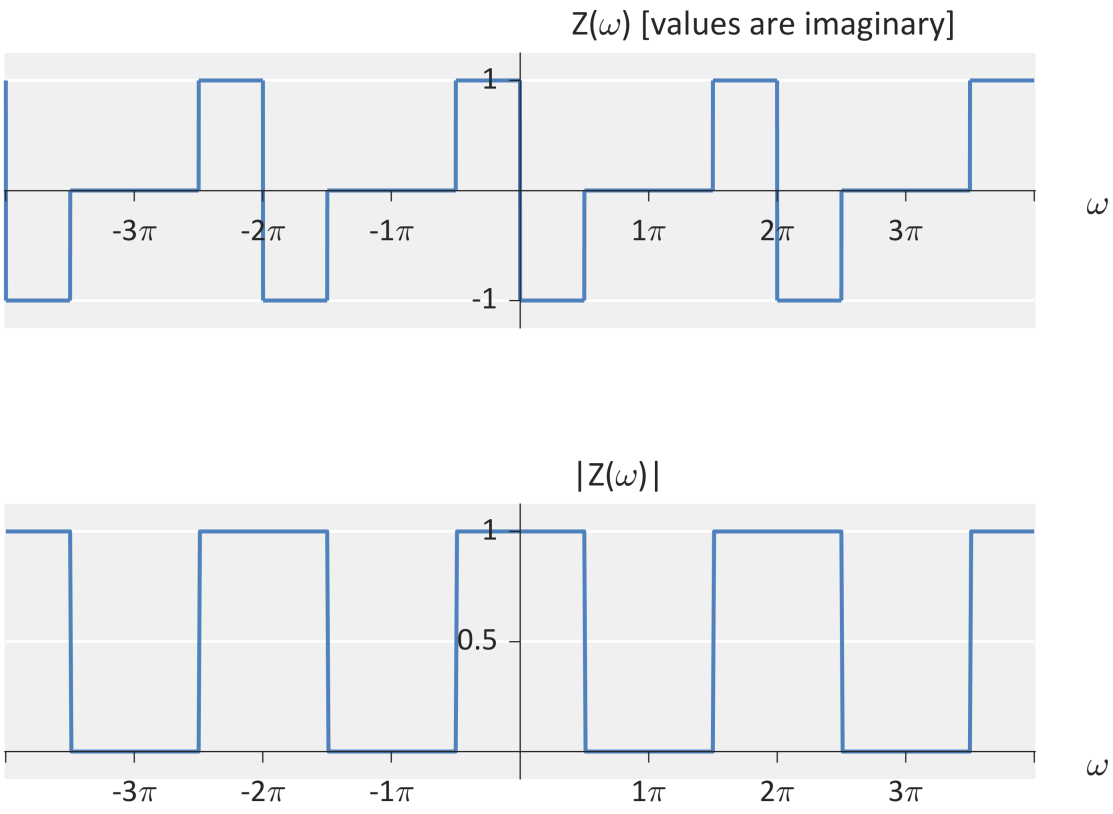
EEE 5502 (Fall 2022) – Practice **Frequency/Magnitude/Phase Responses, 2022**

Question #1: Consider the discrete-time Fourier transform response

$$Z(\omega) = j \sum_{k=-\infty}^{\infty} u(\omega + \pi/2 - 2\pi k) - 2u(\omega - 2\pi k) + u(\omega - \pi/2 - 2\pi k)$$

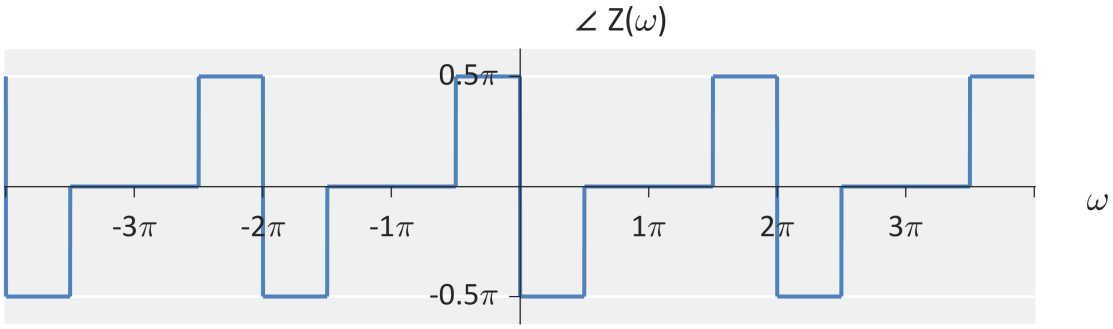
(a) Sketch the magnitude of $Z(\omega)$ (i.e., $|Z(\omega)|$) for $-4\pi \leq \omega \leq 4\pi$.

Solution:



(b) Sketch the phase $Z(\omega)$ (i.e., $\angle Z(\omega)$) for $-4\pi \leq \omega \leq 4\pi$.

Solution:



(c) Is this an FIR filter or an IIR filter?

Solution: IIR

(d) Would you describe $Z(\omega)$ as a low pass filter, band pass filter, high pass filter, or none?

Solution: Low pass

Question #2: Consider the discrete-time difference equation

$$y[n] = y[n-3] - x[n]$$

(a) Determine the transfer function $H(z)$ for the system.

Solution:

$$\begin{aligned} Y(z) &= Y(z)z^{-3} - X(z) \\ Y(z)[1 - z^{-3}] &= -X(z) \\ H(z) &= \frac{-1}{1 - z^{-3}} \end{aligned}$$

(b) Determine the frequency response $H(\omega)$ for the system.

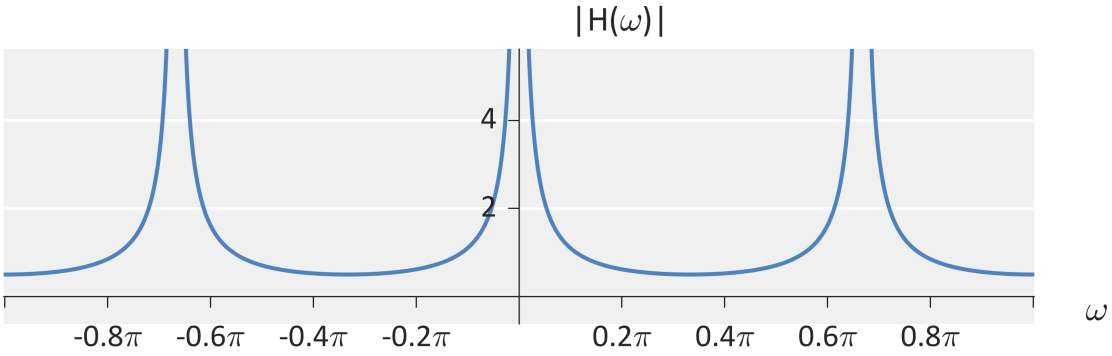
Solution:

$$\begin{aligned} H(\omega) &= \frac{-1}{1 - e^{-j3\omega}} \\ &= \frac{-1}{\left(e^{+j(3/2)\omega} - e^{-j(3/2)\omega}\right) e^{-j(3/2)\omega}} \\ &= \frac{-e^{j(3/2)\omega}}{e^{+j(3/2)\omega} - e^{-j(3/2)\omega}} \\ &= \frac{-e^{j(3/2)\omega}}{2j\sin((3/2)\omega)} \end{aligned}$$

(c) Compute and sketch the magnitude of $H(\omega)$ (i.e., $|H(\omega)|$) for $-\pi \leq \omega \leq \pi$.

Solution:

$$|H(\omega)| = \frac{1}{2|\sin((3/2)\omega)|}$$

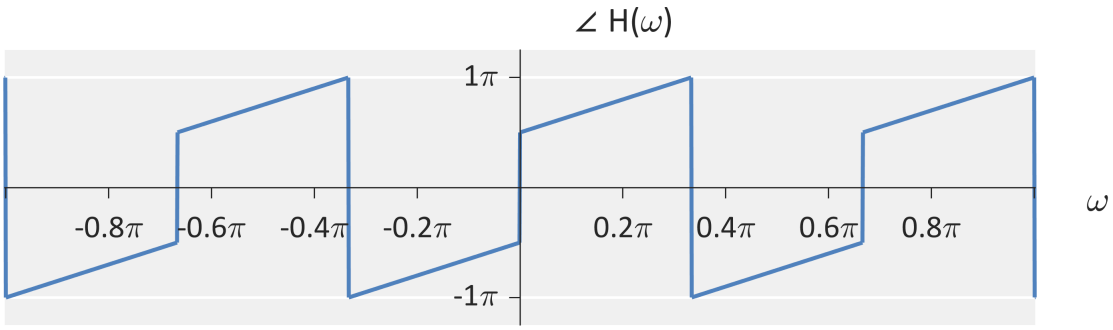


(d) Compute and sketch the phase $H(\omega)$ (i.e., $\angle H(\omega)$) for $-\pi \leq \omega \leq \pi$..

Solution:

$$\begin{aligned} H(\omega) &= \frac{je^{j(3/2)\omega}}{2\sin((3/2)\omega)} \\ &= \frac{e^{j(\pi/2)}e^{j(3/2)\omega}}{2\sin((3/2)\omega)} \\ &= \frac{e^{j((3/2)\omega+\pi/2)}}{2\sin((3/2)\omega)} \end{aligned}$$

$$\angle H(\omega) = \begin{cases} (3/2)\omega + \pi/2 & \text{when } \sin((2/3)\omega) > 0 \\ (3/2)\omega - \pi/2 & \text{when } \sin((2/3)\omega) < 0 \end{cases}$$



(e) Is this an FIR filter or an IIR filter?

Solution: IIR

(f) Would you describe this filter as a low-pass, band-pass, high-pass, or none-of-the-above filter?

Solution: None-of-the-above (it is a combination of a low-pass and band-pass filter)