Full Name: ______________________________

EEE 5502 (Fall 2022) – Practice **Fourier Transform Properties, 2022**

Question #1: Prove the following for a signal $x(t)$ and its Fourier Transform $X(\Omega)$.

(a) If $x(t)$ is real, then $X(\Omega) = X^*(-\Omega)$, where $(\cdot)^*$ is the complex conjugate

(b) If $x(t)$ is real and even, then $X(\omega)$ is real and even
(hint: $X(\Omega)$ is even if $X(\Omega) = X(-\Omega)$ and $X(\omega)$ is real if $X(\Omega) = X^*(\Omega)$)

(c) If $x(t)$ is real and odd, then $X(\Omega)$ is imaginary and odd

Question #2: Use the definitions of the discrete-time Fourier transform (DTFT) / inverse DTFT to answer the questions below. As a reminder, the DTFT / inverse DTFT is defined by

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \quad , \quad x[n] = \frac{1}{2\pi}\int_{2\pi} X(\omega)e^{j\omega n}\, d\omega$$

(a) Show that the DTFT of a signal is always periodic with a period of 2π.

(b) Show that if $x[n]$ is real, then its DTFT $X(\omega)$ has conjugate symmetry:

$$X(\omega) = X^*(-\omega)$$

(c) Show that if $x[n]$ is real (the complex case is not much more difficult), then

$$X_e(\omega) = \sum_{n=-\infty}^{\infty} x_e[n]\cos(\omega n) \quad , \quad X_o(\omega) = -j\sum_{n=-\infty}^{\infty} x_o[n]\sin(\omega n)$$

where $x_e[n]$ and $x_o[n]$ are the even and odd parts of $x[n]$, respectively.

Question #3: Rectangular functions are used throughout signal processing and other fields (e.g., an aperture of a camera in two-dimensions can be represented by a rectangle and its effect on images is a convolution). Hence, the CTFT or DTFT of the rectangular function is also commonly used. In this problem, let's derive the DTFT of a rectangular function. That is, show that following DTFT pair is true:

$$x[n] = u[n+N] - u[n-N-1]$$
$$X(\omega) = \frac{\sin(\omega(N+1/2))}{\sin(\omega/2)}$$

You may use the DTFT tables to show this, except do not use the row of the table that explicitly gives this relationship. [1]

[1] Hint: You may want to use something similar to $1 - e^{-j\omega} = e^{-j\omega/2}(e^{+j\omega/2} - e^{-j\omega/2})$