Full Name: _____

EEE 5502 (Fall 2022) – Practice **Poles and Zeros / Causality / Stability, 2022**

Question #1: For each of the following z-transforms,

- ***Do not*** compute the inverse Z-transform
- Plot the pole-zero plot and the region of convergence
- Answer if the system is stable or unstable

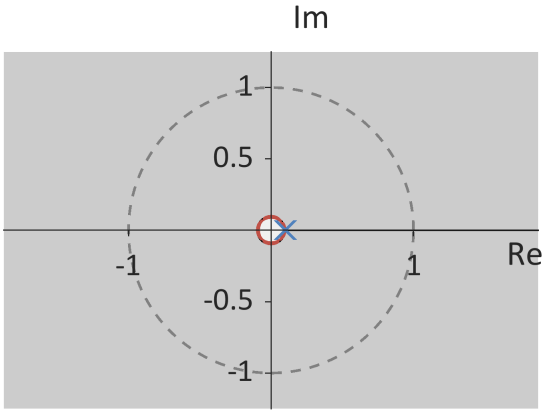
Use the discrete-time transform tables on the course website.

(a) $H(z) = \dfrac{1}{1-(1/10)z^{-1}}$ (assume the system is *causal*)

Solution:

$$H(z) = \frac{1}{1-(1/10)z^{-1}} = \frac{z}{1-(1/10)}$$

Zeros: $z = 0$, Poles: $z = 1/10$
Stable

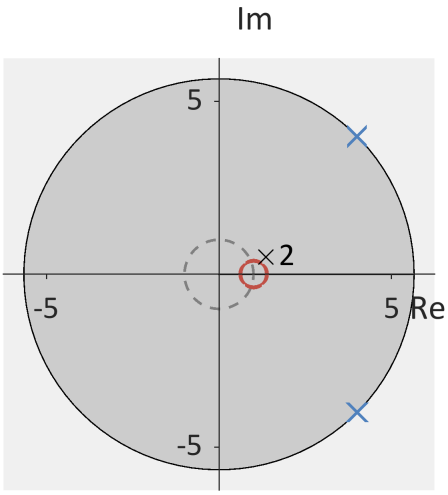


(b) $H(z) = \dfrac{z^2-2z+1}{(z-4)^2+16}$ (assume the system is *anti-causal*)

Solution:

$$H(z) = \frac{z^2 - 2z + 1}{(z-4)^2 + 16}$$
$$= \frac{(z-1)^2}{(z-4+4j)(z-4-4j)}$$

Zeros: $z = 1, 1$, Poles: $z = 4 - 4j, 4 + 4j$
Stable



(c) $H(z) = \dfrac{1}{1-(1/4)z^{-1}} + \dfrac{4}{1+2z^{-1}}$ (assume the system is *stable*)

Solution:

$$H(z) = \frac{1}{1-(1/4)z^{-1}} + \frac{4}{1+2z^{-1}}$$
$$= \frac{(1+2z^{-1}) + 4(1-(1/4)z^{-1})}{(1-(1/4)z^{-1})(1+2z^{-1})}$$
$$= \frac{1+2z^{-1}+4-z^{-1}}{(1-(1/4)z^{-1})(1+2z^{-1})}$$
$$= \frac{5+z^{-1}}{(1-(1/4)z^{-1})(1+2z^{-1})}$$
$$= \frac{1}{5}\,\frac{1+1/5z^{-1}}{(1-(1/4)z^{-1})(1+2z^{-1})}$$
$$= \frac{1}{5}\,\frac{z(z+(1/5))}{(z-(1/4))(z+2)}$$

Zeros: $z = 0, -1/5$, Poles: $z = 1/4, -2$
Stable