Full Name: _____

EEE 5502 (Fall 2022) – Practice **The Inverse Z-Transform, 2022**

Question #1: Determine the impulse responses $h[n]$ for the following difference equations.

(a) $y[n] + (0.8)y[n-1] = x[n]$ (Assume system is causal)

Solution:

$$\begin{aligned} Y(z) + 0.8Y(z)z^{-1} &= X(z) \\ Y(z)[1 + 0.8z^{-1}] &= X(z) \\ H(z) &= \frac{1}{1 + 0.8z^{-1}} \\ h[n] &= (-0.8)^n u[n] \end{aligned}$$

(b) $y[n] - 4y[n-1] + 4y[n-2] = x[n]$ (Assume system is anti-causal)

Solution:

$$\begin{aligned} Y(z) - 4Y(z)z^{-1} + 4Y(z)z^{-2} &= X(z) \\ Y(z)[1 - 4z^{-1} + 4z^{-2}] &= X(z) \\ H(z) &= \frac{1}{1 - 4z^{-1} + 4z^{-2}} \\ &= \frac{1}{(1 - 2z^{-1})^2} \\ &= (1/2)z \frac{2z^{-1}}{(1 - 2z^{-1})^2} \\ &= -(1/2)(n+1)a^{n+1}u[-(n+1) - 1] \\ &= -(1/2)(n+1)a^{n+1}u[-n - 2] \end{aligned}$$

(c) $y[n] - 2.5y[n-1] + y[n-2] = x[n]$ (Assume system is causal)

Solution:

$$
\begin{aligned}
Y(z)[1 - 2.5z^{-1} + z^{-2}] &= X(z) \\
H(z) &= \frac{1}{1 - 2.5z^{-1} + z^{-2}} \\
&= \frac{1}{(1 - 2z^{-1})(1 - 0.5z^{-1})} \\
&= \frac{4/3}{1 - 2z^{-1}} - \frac{1/3}{1 - (1/2)z^{-1}} \\
&= (4/3)(2)^n u[n] - (1/3)(1/2)^n u[n]
\end{aligned}
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