Full Name: ____________________

EEE 5502 (Fall 2022) – Practice **The Z-Transform, 2022**

Question #1: Consider the difference equation

$$y[n] - \cos(\omega_0)y[n-1] + (1/4)y[n-2] = x[n] \ ,$$

which represents a causal LTI system.

(a) Compute the z-transform of the system $H(z)$.

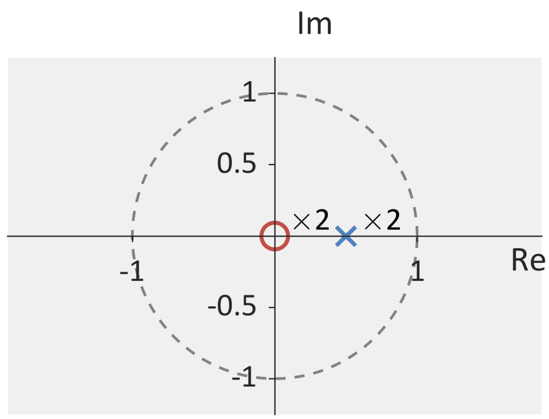
Solution:

$$\begin{aligned} Y(z) - \cos(\omega_0)Y(z)z^{-1} + (1/4)Y(z)z^{-2} &= X(z) \\ Y(z)[1 - \cos(\omega_0)z^{-1} + (1/4)z^{-2}] &= X(z) \\ H(z) = \frac{Y(z)}{X(z)} &= \frac{1}{1 - \cos(\omega_0)z^{-1} + (1/4)z^{-2}} \end{aligned}$$

(b) Sketch the pole-zero plot for the system for $\omega_0 = 0$, $\omega_0 = \pi/3$, and $\omega_0 = \pi/2$.

Solution:

$$\begin{aligned} H(z) &= \frac{z^2}{z^2 - \cos(\omega_0)z + (1/4)} \\ H(z) &= \frac{z^2}{z^2 - (1/2)\left[e^{+j\omega_0} + e^{-j\omega_0}\right]z + (1/4)} \\ H(z) &= \frac{z^2}{(z - (1/2)e^{+j\omega_0})(z - (1/2)e^{-j\omega_0})} \end{aligned}$$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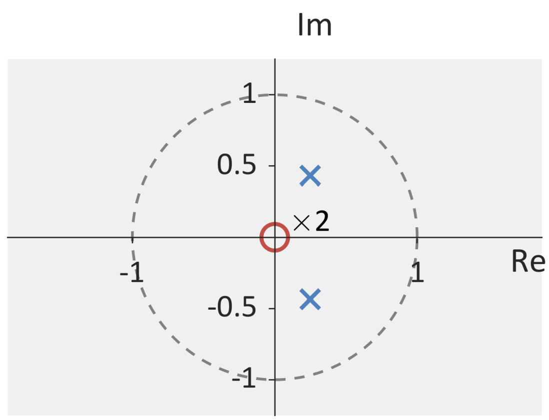
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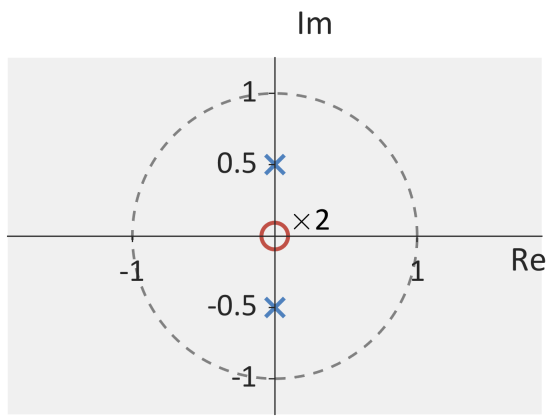
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Question #2: Consider the impulse response

$$h[n] = (2)^{n-1}u[n-2] - (1/4)^{-n}u[-n] \ ,$$

which represents a causal LTI system.

(a) Compute the z-transform of the system $H(z)$.

Solution:

$$\begin{aligned}
h[n] &= (2)^{n-1}u[n-2] - (1/4)^{-n}u[-n] \\
&= 2^{+1}(2)^{n-2}u[n-2] - (1/4)^{-n}u[-n] \\
\mathcal{Z}\left\{(2)^{n-1}u[n-2]\right\} &= 2\frac{z^{-2}}{1-2z^{-1}} \\
\mathcal{Z}\left\{(1/4)^{-n}u[-n]\right\} &= \frac{1}{1-(1/4)z} \\
&= \frac{z^{-1}}{z^{-1}-(1/4)} \\
&= \frac{-z^{-1}}{(1/4)-z^{-1}} \\
&= \frac{-4z^{-1}}{1-4z^{-1}} \\
H(z) &= 2\frac{z^{-2}}{1-2z^{-1}} + \frac{4z^{-1}}{1-4z^{-1}}
\end{aligned}$$

(b) Sketch the pole-zero plot for the system with transfer function

$$H(z) = \frac{2}{z^{-1}} + \frac{1+z^{-1}}{1-0.5z^{-1}}$$

Solution:

$$
\begin{aligned}
H(z) &= \frac{2}{z^{-1}} + \frac{1+z^{-1}}{1-0.5z^{-1}} \\
&= \frac{2(1-0.5z^{-1})}{z^{-1}(1-0.5z^{-1})} + \frac{z^{-1}(1+z^{-1})}{z^{-1}(1-0.5z^{-1})} \\
&= \frac{2-z^{-1}+z^{-1}+z^{-2}}{z^{-1}(1-0.5z^{-1})} \\
&= \frac{2+z^{-2}}{z^{-1}(1-0.5z^{-1})} \\
&= 2\,\frac{1+0.5z^{-2}}{z^{-1}(1-0.5z^{-1})}
\end{aligned}
$$

To compute the poles and zeros, let's make this a function of z rather than z^{-1}

$$
\begin{aligned}
H(z) &= 2\,\frac{1+0.5z^{-2}}{z^{-1}(1-0.5z^{-1})} \\
&= 2\,\frac{z^{-2}}{z^{-2}}\,\frac{z^2+0.5}{z-0.5} \\
&= 2\,\frac{z^2+0.5}{z-0.5}
\end{aligned}
$$

So there is a single pole at $z = 0.5$ and two zeros at $z = (1/\sqrt{2})j, -(1/\sqrt{2})j$.