

Question #1: Consider the following discrete-time systems. Assume M is a positive integer. Assume the input to every system is $x[n]$ (i.e., the $y[n] = \mathcal{H}\{x[n]\}$).

$$y_1[n] = x[n - M]$$

$$y_2[n] = \frac{1}{M} \sum_{m=0}^{M-1} x[n - m]$$

$$y_3[n] = \frac{1}{M} \sum_{m=1}^M x[n + m]$$

$$y_4[n] = \sum_{m=-\infty}^n |x[m]|^2$$

$$y_5[n] = 0.5y_5[n - M] + x[n]$$

$$y_6[n] = x[Mn]$$

(a) Identify a non-linear system. Show that it is non-linear.

(b) Identify a time-varying system. Show that it is time-varying.

(c) Identify a non-causal system. Show that it is non-causal.

(d) Identify a memoryless system. Show that it is memoryless.

(e) Identify a non-BIBO stable system. Show that it is not BIBO stable

Question #2: Consider the following discrete-time systems. Assume $M = 4$ is a positive integer. Assume the input to every system is $x[n]$ (i.e., the $y[n] = \mathcal{H}\{x[n]\}$).

$$\begin{aligned}
 y_1[n] &= x[n - M] & y_2[n] &= \frac{1}{M} \sum_{m=0}^{M-1} x[n - m] \\
 y_3[n] &= \frac{1}{M} \sum_{m=1}^M x[n + m] & y_4[n] &= \sum_{m=-\infty}^n |x[n]|^2 \\
 y_5[n] &= 0.5y[n - M] + x[n] & y_6[n] &= x[Mn]
 \end{aligned}$$

Sketch the output of each system for input $x[n] = u[n] - u[n - 3]$ for $-5 \leq n \leq 11$.