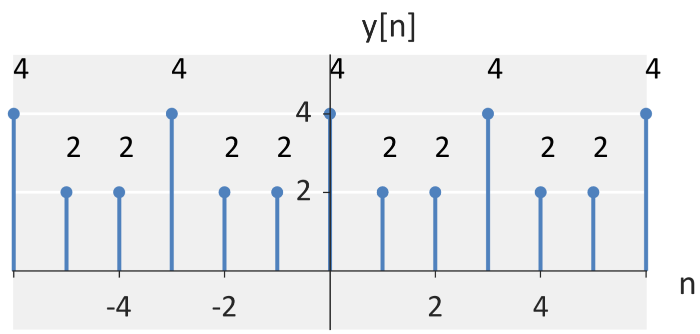
Full Name: ____________________

EEE 5502 (Fall 2022) – Practice **Continuous and Discrete-Time Signals, 2022**

Question #1: Let $x[n] = 2\left(\sum_{k=-\infty}^{\infty} u[n-3k] - u[n-4-3k]\right)$ be a discrete-time signal.

(a) Sketch the signal $x[n]$ for $-6 \leq n < 6$.

Solution:



(b) Is $x[n]$ an infinite length signal or finite length signal?

Solution: Infinite Length

(c) Is $x[n]$ signal periodic? If so, determine the fundamental period.

Solution: Periodic with period $N = 3$

(d) Is $x[n]$ an even signal, odd signal, or neither?

Solution: Even

(e) Is $x[n]$ causal, anti-causal, or neither?

Solution: Neither

(f) Compute the energy and power of $x[n]$ [for *all* time].

Solution: Energy: $E_x = \infty$
Power: $P_x = \frac{1}{3}\left(2^2 + 2^2 + 4^2\right) = 24/3 = 8$

Question #2: Let $x[n] = j^{n/123}$ be a discrete-time signal.

(a) Is $x[n]$ an infinite length signal or finite length signal?

Solution: Infinite Length

(b) Is $x[n]$ signal periodic? If so, determine the fundamental period.

Solution:

$$j = e^{j\pi/2}$$

So

$$j^{n/123} = e^{jn(\pi/246)}$$

The fundamental period occurs when

$$(\pi/246)N_0 = 2\pi$$
$$N_0 = 492$$

Periodic with fundamental period $N_0 = 492$

(c) Is $x[n]$ an even signal, odd signal, or neither?

Solution: Real part is even, imaginary part is odd

(d) Is $x[n]$ causal, anti-causal, or neither?

Solution: Neither

(e) Compute the energy and power of $x[n]$ [for *all* time].

Solution: Energy: $E_x = \infty$
Power: $P_x = \frac{1}{492}\sum_{n=0}^{491} |e^{j(\pi/246)n}|^2 = 1$

Question #3: Answer the following questions.

(a) Let $x_e[n]$ be an even signal and let $x_o[n]$ be an odd signal. Show that

$$x_e[n] = \frac{1}{2}\left(x_e[n] + x_e[-n]\right)$$
$$x_o[n] = \frac{1}{2}\left(x_o[n] - x_o[-n]\right)$$

Solution: Since $x_o[n] = -x_o[-n]$ and $x_e[n] = x_e[-n]$, we can rewrite the above expressions

$$\begin{aligned}\frac{1}{2}\left(x_e[n] + x_e[-n]\right) &= \frac{1}{2}\left(x_e[n] + x_e[n]\right) \\ &= x_e[n]\end{aligned}$$

$$\begin{aligned}\frac{1}{2}\left(x_o[n] - x_o[-n]\right) &= \frac{1}{2}\left(x_o[n] + x_o[n]\right) \\ &= x_o[n]\end{aligned}$$

(b) Let $x_e[n]$ be an even signal and let $x_o[n]$ be an odd signal. Show that

$$\sum_{n=-\infty}^{\infty} x_e[n]x_o[n] = 0 \ .$$

Solution: Since $x_o[n] = -x_o[-n]$ and $x_e[n] = x_e[-n]$, we can rewrite the above expression

$$\begin{aligned}\sum_{n=-\infty}^{\infty} x_e[n]x_o[n] &= x_e[0]x_o[0] + \sum_{n=1}^{\infty} x_e[n]x_o[n] + \sum_{n=-\infty}^{-1} x_e[n]x_o[n] \\ &= x_e[0](0) + \sum_{n=1}^{\infty} x_e[n]x_o[n] + \sum_{n=1}^{\infty} x_e[-n]x_o[-n] \\ &= \sum_{n=1}^{\infty} x_e[n]x_o[n] - \sum_{n=1}^{\infty} x_e[n]x_o[n] \\ &= 0 \ .\end{aligned}$$

(c) Show that $x[n] = \cos(\pi^2 n)$ is not periodic.

Solution: The fundamental period N of a cosine is defined as any smallest multiple of N such that $\pi^2 N = 2\pi$ that is an integer. So,

$$N = 2/\pi \ .$$

However, this is an irrational number. As a result, its multiple will never be an integer.