Full Name: ______________________ **ExamID: 010001**

EEL 4750 / EEE 5502 (Fall 2019) – Practice Exam #03 **Date: Nov. 29, 2019**

Question	# of Points Possible	# of Points Obtained	Grader
# 1	17		
# 2	15		
# 3	16		
# 4	18		
# 5	18		
# 6	16		
Total	100		

For full credit when sketching: remember to label axes and make locations and amplitudes clear.

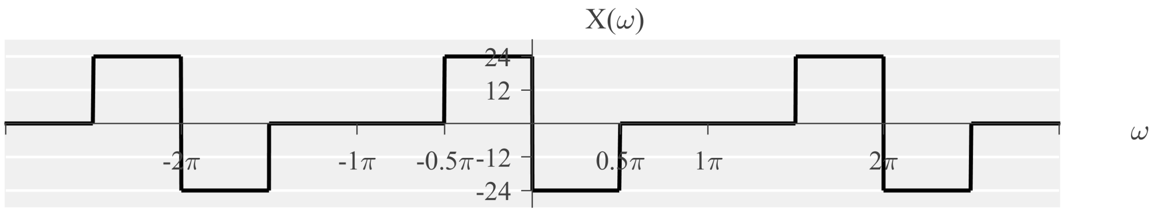
Before starting the exam, read and sign the following agreement.
By signing this agreement, I agree to solve the problems of this exam while adhering to the policies and guidelines of the University of Florida and EEL 4750 / EEE 5502 and without additional external help. The guidelines include, but are not limited to,

- The University of Florida honor pledge: "On my honor, I have neither given nor received unauthorized aid in doing this assignment."
- Only one 8.5 by 11 inch cheat sheet (double-sided) may be used
- No calculators or computers may be used
- No textbooks or additional notes may be used
- No collaboration is allowed
- No cheating is allowed

______________________ ______________________

Student Date

Question #1: Consider the DTFT of the signal $x[n]$ (i.e., $X(\omega)$) shown below.

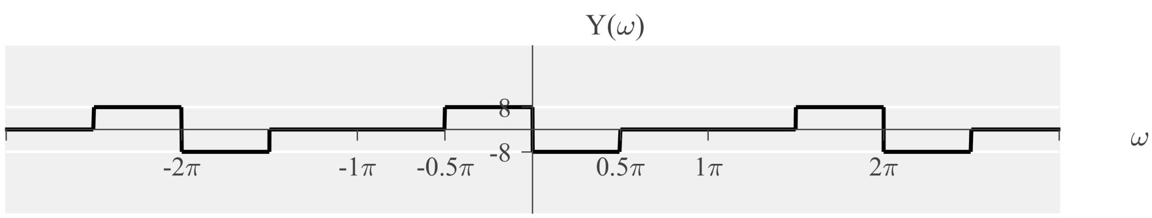


(a) *(4 pts)* What is the maximum achievable downsampling factor for $5x[n]$ without aliasing?

Solution: The maximum downsampling factor is 2

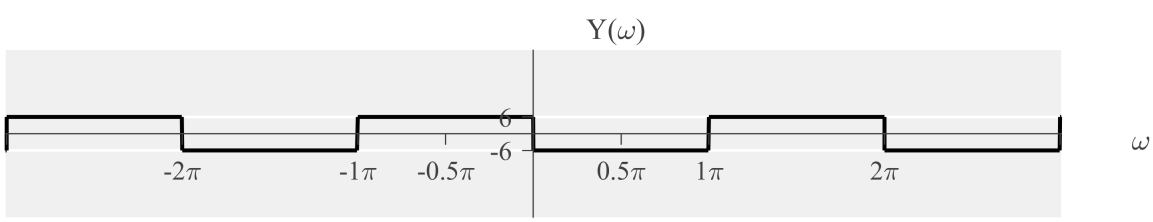
(b) *(7 pts)* Sketch the DTFT (from $\omega = -3\pi$ to $\omega = 3\pi$) of $x[n]$ after downsampling by 3 (**with no anti-aliasing filter**). Remember to label important locations / values.

Solution:

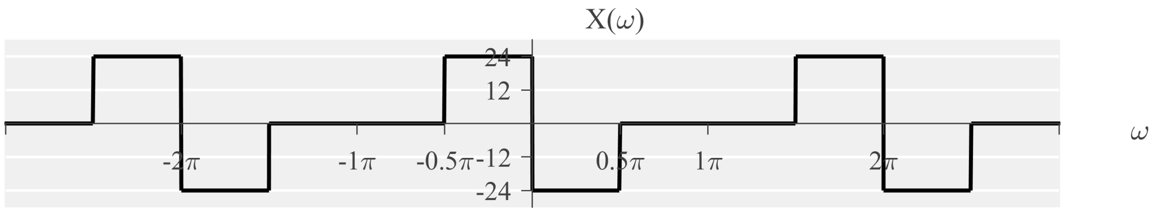


(c) *(8 pts)* Sketch the DTFT (from $\omega = -3\pi$ to $\omega = 3\pi$) of $x[n]$ after downsampling by 4 (**with an anti-aliasing filter**). Remember to label important locations / values.

Solution:

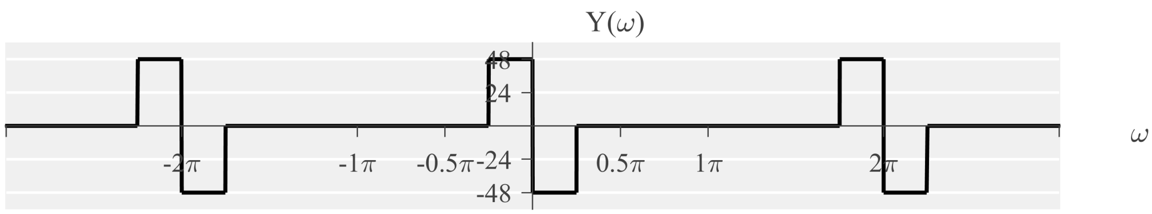


Question #2: Consider the DTFT of the signal $x[n]$ (i.e., $X(\omega)$) shown below.



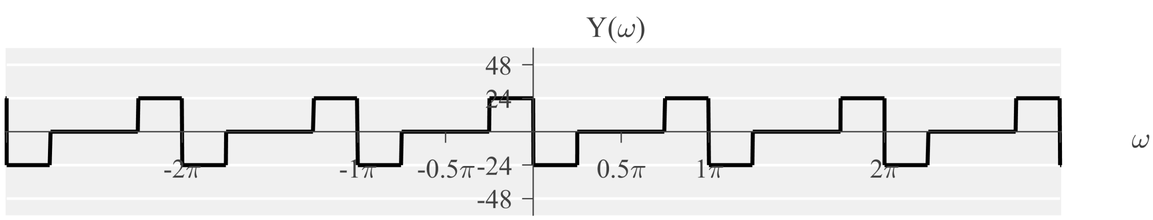
(a) *(8 pts)* Sketch the DTFT (from $\omega = -3\pi$ to $\omega = 3\pi$) of $x[n]$ after upsampling by 2 (**with an interpolation filter**). Remember to label important locations / values.

Solution:

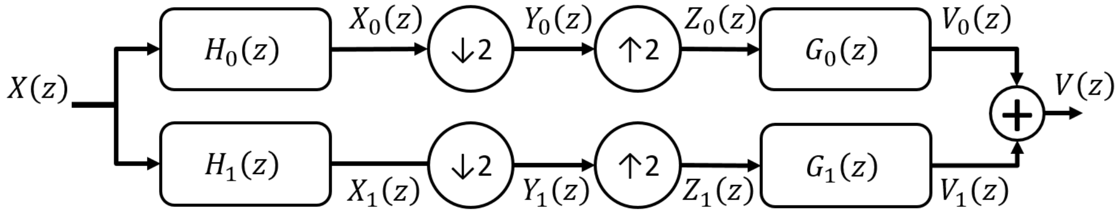


(b) *(7 pts)* Sketch the DTFT (from $\omega = -3\pi$ to $\omega = 3\pi$) of $x[n]$ after upsampling by 2 (**with no interpolation filter**). Remember to label important locations / values.

Solution:



Question #3: Consider a 2-channel filter bank shown below.



Let the filters be defined by the frequency domain expression

$$H_0(\omega) = G_0(\omega) = \sqrt{2}\sin(\omega/2)$$

(a) *(7 pts)* Choose a filter $H_1(\omega) = G_1(\omega)$ that satisfies the alias canceling conditions.

Solution: The alias canceling conditions:

$$H_0(\omega)G_0(\omega) + H_1(\omega)G_1(\omega) = 2$$
$$H_0(\omega-\pi)G_0(\omega) + H_1(\omega-\pi)G_1(\omega) = 0$$

$$2\sin(\omega/2)\sin(\omega/2) + H_1(\omega)G_1(\omega) = 2$$
$$2\sin((\omega-\pi)/2)\sin(\omega/2) + H_1(\omega-\pi)G_1(\omega) = 0$$

$$2\sin^2(\omega/2) + H_1(\omega)G_1(\omega) = 2$$
$$\text{-}2\cos(\omega/2)\sin(\omega/2) + H_1(\omega-\pi)G_1(\omega) = 0$$

If we choose $H_1(\omega) = G_1(\omega) = \sqrt{2}\cos(\omega/2)$,

$$2\sin^2(\omega/2) + 2\cos^2(\omega/2) = 2$$
$$\text{-}2\cos(\omega/2)\sin(\omega/2) + 2\cos((\omega-\pi)/2)\cos(\omega/2) = 0$$

$$2\sin^2(\omega/2) + 2\cos^2(\omega/2) = 2$$
$$\text{-}2\cos(\omega/2)\sin(\omega/2) + 2\sin(\omega/2)\cos(\omega/2) = 0$$

$$2 = 2$$
$$0 = 0$$

(b) *(7 pts)* Let $X(\omega) = \cos(\omega/2)$. Compute the intermediate signal $V_0(\omega)$.

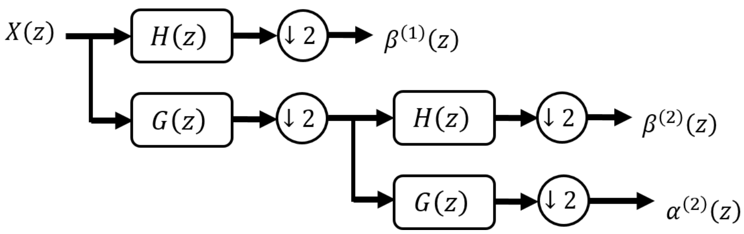
Solution: The frequency response at $V_0(\omega)$ is

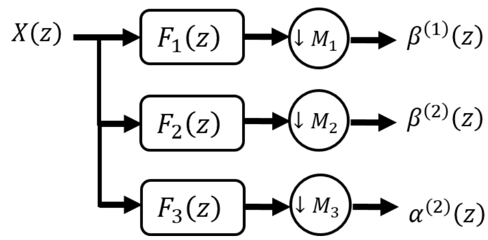
$$\begin{aligned} V_0(\omega) &= (1/2)\left[H_0(\omega)X(\omega) + H_0(\omega-\pi)X(\omega-\pi)\right] G_0(\omega) \\ &= (1/2)\left[\sin(\omega/2)\cos(\omega/2) + \sin((\omega-\pi)/2)\cos((\omega-\pi)/2)\right] G_0(\omega) \\ &= (1/2)\left[\sin(\omega/2)\cos(\omega/2) - \cos(\omega/2)\sin(\omega/2)\right] G_0(\omega) \\ &= 0 \end{aligned}$$

(c) *(4 pts)* **(True or False)** When the alias canceling conditions are met, $V_0(z) = V_1(z)$.

Solution: **False**, alias canceling ensures that $V_0(z) + V_1(z) = X(z)$, which is not guaranteed to be true when $V_0(z) = V_1(z)$.

Question #4: Consider the following wavelet bank and filter bank.





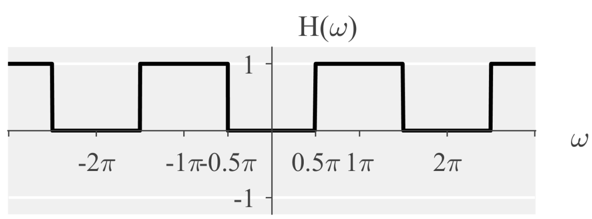
Let the high pass filter $H(z)$ and low pass filter $G(z)$ be defined by frequency responses:

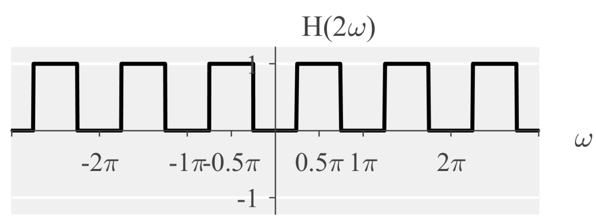
$$G(\omega) = \sum_{k=-\infty}^{\infty} u(\omega + \pi/2 - 2\pi k) - u(\omega - \pi/2 - 2\pi k)$$
$$H(\omega) = \sum_{k=-\infty}^{\infty} u(\omega + \pi/2 - \pi - 2\pi k) - u(\omega - \pi/2 - \pi - 2\pi k)$$

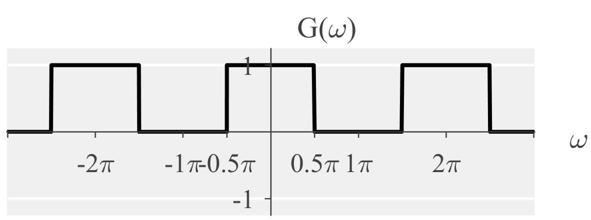
Use the Noble identities to simplify the wavelet bank (left) diagram and represent it as a filter bank (right). Determine M_1, M_2, and M_3. Sketch $|F_1(\omega)|$, $|F_2(\omega)|$, and $|F_3(\omega)|$.

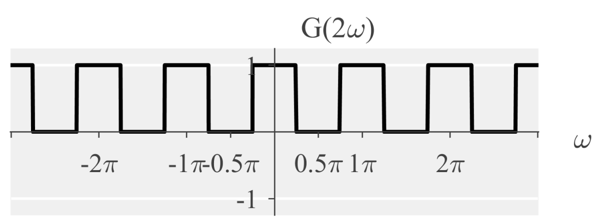
Solution: $M_1 = 2,\ M_2 = 4,\ M_3 = 4.$

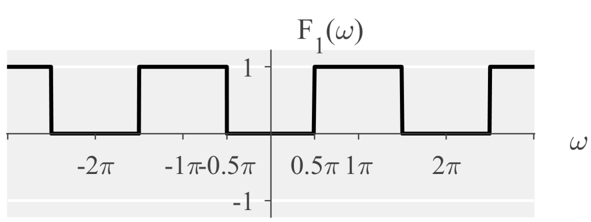
$$F_1(\omega) = H(z)$$
$$F_2(\omega) = G(z)H(z^2)$$
$$F_3(\omega) = G(z)G(z^2)$$

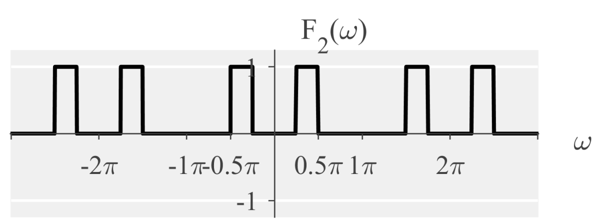












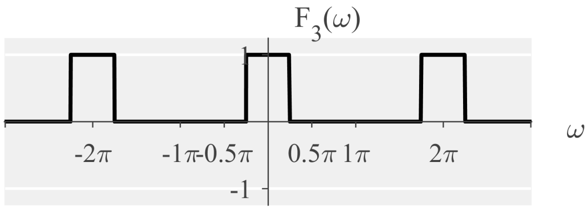

$F_3(\omega)$
1
-1
-2π
-1π
-0.5π
0.5π
1π
2π
ω

Table of Discrete-Time Fourier Transform Pairs:

$$\text{Discrete-Time Fourier Transform} \quad : \quad X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$\text{Inverse Discrete-Time Fourier Transform} \quad : \quad x[n] = \frac{1}{2\pi} \int_{2\pi} X(\omega) e^{j\omega t}\, d\omega \; .$$

$x[n]$	$X(\omega)$	condition
$a^n u[n]$	$\frac{1}{1 - ae^{-j\omega}}$	$\lvert a\rvert < 1$
$(n+1)a^n u[n]$	$\frac{1}{(1 - ae^{-j\omega})^2}$	$\lvert a\rvert < 1$
$\frac{(n+r-1)!}{n!(r-1)!} a^n u[n]$	$\frac{1}{(1 - ae^{-j\omega})^r}$	$\lvert a\rvert < 1$
$\delta[n]$	1	
$\delta[n - n_0]$	$e^{-j\omega n_0}$	
$x[n] = 1$	$2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$	
$u[n]$	$\frac{1}{1 - e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega - 2\pi k)$	
$e^{j\omega_0 n}$	$2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - \omega_0 - 2\pi k)$	
$\cos(\omega_0 n)$	$\pi \sum_{k=-\infty}^{\infty} \{\delta(\omega - \omega_0 - 2\pi k) + \delta(\omega + \omega_0 - 2\pi k)\}$	
$\sin(\omega_0 n)$	$\frac{\pi}{j} \sum_{k=-\infty}^{\infty} \{\delta(\omega - \omega_0 - 2\pi k) - \delta(\omega + \omega_0 - 2\pi k)\}$	
$\sum_{k=-\infty}^{\infty} \delta[n - kN]$	$\frac{2\pi}{N} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{N}\right)$	
$x[n] = \begin{cases} 1 \;, & \lvert n\rvert \le N \\ 0 \;, & \lvert n\rvert > N \end{cases}$	$\frac{\sin(\omega(N + 1/2))}{\sin(\omega/2)}$	
$\frac{\sin(Wn)}{\pi n} = \frac{W}{\pi}\mathrm{sinc}\left(\frac{Wn}{\pi}\right)$	$X(\omega) = \begin{cases} 1 \;, & 0 \le \lvert\omega\rvert \le W \\ 0 \;, & W < \lvert\omega\rvert \le \pi \end{cases}$ $X(\omega)$ is periodic with period 2π	

Table of Discrete-Time Fourier Transform Properties: For each property, assume

$$x[n] \overset{DTFT}{\longleftrightarrow} X(\omega) \quad \text{and} \quad y[n] \overset{DTFT}{\longleftrightarrow} Y(\omega)$$

Property	Time domain	DTFT domain
Linearity	$Ax[n] + By[n]$	$AX(\omega) + BY(\omega)$
Time Shifting	$x[n - n_0]$	$X(\omega)e^{-j\omega n_0}$
Frequency Shifting	$x[n]e^{j\omega_0 n}$	$X(\omega - \omega_0)$
Conjugation	$x^*[n]$	$X^*(-\omega)$
Time Reversal	$x[-n]$	$X(-\omega)$
Convolution	$x[n] * y[n]$	$X(\omega)Y(\omega)$
Multiplication	$x[n]y[n]$	$\dfrac{1}{2\pi}\displaystyle\int_{2\pi} X(\theta)Y(\omega - \theta)d\theta$
Differencing in Time	$x[n] - x[n-1]$	$(1 - e^{-j\omega})X(\omega)$
Accumulation	$\sum_{k=-\infty}^{\infty} x[k]$	$\frac{1}{1-e^{-j\omega}} + \pi X(0)\sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$
Frequency Differentiation	$nx[n]$	$j\dfrac{dX(\omega)}{d\omega}$
Parseval's Relation for Aperiodic Signals	$\sum_{k=-\infty}^{\infty} \|x[k]\|^2$	$\frac{1}{2\pi}\int_{2\pi} \|X(\omega)\|^2 d\omega$

Table of Z-Transform Pairs:

$$\text{Z-Transform} \quad : \quad X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$\text{Inverse Z-Transform} \quad : \quad x[n] = \frac{1}{2\pi j} \oint_{\mathcal{C}} X(z) z^{n-1}\, dz \, .$$

$x[n]$	$X(\omega)$	ROC
$a^n u[n]$	$\frac{1}{1 - az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u[-n-1]$	$\frac{1}{1 - az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$na^n u[n]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
$\delta[n]$	1	All z
$\delta[n - n_0]$	z^{-n_0}	All z
$u[n]$	$\frac{1}{1 - z^{-1}}$	$\lvert z\rvert > 1$
$\cos(\omega_0 n) u[n]$	$\frac{1 - z^{-1}\cos(\omega_0)}{1 - 2z^{-1}\cos(\omega_0) + z^{-2}}$	$\lvert z\rvert > 1$
$\sin(\omega_0 n) u[n]$	$\frac{z^{-1}\sin(\omega_0)}{1 - 2z^{-1}\cos(\omega_0) + z^{-2}}$	$\lvert z\rvert > 1$
$a^n \cos(\omega_0 n) u[n]$	$\frac{1 - az^{-1}\cos(\omega_0)}{1 - 2az^{-1}\cos(\omega_0) + a^2 z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$
$a^n \sin(\omega_0 n) u[n]$	$\frac{az^{-1}\sin(\omega_0)}{1 - a2z^{-1}\cos(\omega_0) + a^2 z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$

Table of Z-Transform Properties: For each property, assume

$$x[n] \overset{Z}{\longleftrightarrow} X(z) \quad \text{and} \quad y[n] \overset{Z}{\longleftrightarrow} Y(z)$$

Property	Time domain	Z-domain
Linearity	$Ax[n] + By[n]$	$AX(z) + BY(z)$
Time Shifting	$x[n - n_0]$	$X(z)z^{-n_0}$
Z-scaling	$a^n x[n]$	$X(a^{-1}z)$
Conjugation	$x^*[n]$	$X^*(z^*)$
Time Reversal	$x[-n]$	$X(z^{-1})$
Convolution	$x[n] * y[n]$	$X(z)Y(z)$
Differentiation in z-domain	$nx[n]$	$-z\frac{dX(z)}{dz}$
Initial Value Theorem	$x[n]$ is causal	$x(0) = \lim_{z\to\infty} X(z)$