Table of Discrete-Time Fourier Transform Pairs:

$$\text{Discrete-Time Fourier Transform} \quad : \quad X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$\text{Inverse Discrete-Time Fourier Transform} \quad : \quad x[n] = \frac{1}{2\pi}\int_{2\pi} X(\omega)e^{j\omega n}\, d\omega \ .$$

$x[n]$	$X(\omega)$	condition
$a^n u[n]$	$\frac{1}{1-ae^{-j\omega}}$	$\|a\| < 1$
$(n+1)a^n u[n]$	$\frac{1}{(1-ae^{-j\omega})^2}$	$\|a\| < 1$
$\frac{(n+r-1)!}{n!(r-1)!}a^n u[n]$	$\frac{1}{(1-ae^{-j\omega})^r}$	$\|a\| < 1$
$\delta[n]$	1	
$\delta[n-n_0]$	$e^{-j\omega n_0}$	
$x[n] = 1$	$2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$	
$u[n]$	$\frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega - 2\pi k)$	
$e^{j\omega_0 n}$	$2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - \omega_0 - 2\pi k)$	
$\cos(\omega_0 n)$	$\pi \sum_{k=-\infty}^{\infty} \{\delta(\omega - \omega_0 - 2\pi k) + \delta(\omega + \omega_0 - 2\pi k)\}$	
$\sin(\omega_0 n)$	$\frac{\pi}{j} \sum_{k=-\infty}^{\infty} \{\delta(\omega - \omega_0 - 2\pi k) - \delta(\omega + \omega_0 - 2\pi k)\}$	
$\sum_{k=-\infty}^{\infty} \delta[n-kN]$	$\frac{2\pi}{N} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{N}\right)$	
$x[n] = \begin{cases} 1 & , \quad \|n\| \le N \\ 0 & , \quad \|n\| > N \end{cases}$	$\frac{\sin(\omega(N+1/2))}{\sin(\omega/2)}$	
$\frac{\sin(Wn)}{\pi n} = \frac{W}{\pi}\text{sinc}\,(Wn)$	$X(\omega) = \begin{cases} 1 & , \quad 0 \le \|\omega\| \le W \\ 0 & , \quad W < \|\omega\| \le \pi \end{cases}$	

$X(\omega)$ is periodic with period 2π

Table of Discrete-Time Fourier Transform Properties: For each property, assume

$$x[n] \overset{DTFT}{\longleftrightarrow} X(\omega) \quad \text{and} \quad y[n] \overset{DTFT}{\longleftrightarrow} Y(\omega)$$

Property	Time domain	DTFT domain
Linearity	$Ax[n] + By[n]$	$AX(\omega) + BY(\omega)$
Time Shifting	$x[n - n_0]$	$X(\omega)e^{-j\omega n_0}$
Frequency Shifting	$x[n]e^{j\omega_0 n}$	$X(\omega - \omega_0)$
Conjugation	$x^*[n]$	$X^*(-\omega)$
Time Reversal	$x[-n]$	$X(-\omega)$
Convolution	$x[n] * y[n]$	$X(\omega)Y(\omega)$
Multiplication	$x[n]y[n]$	$\dfrac{1}{2\pi}\displaystyle\int_{2\pi} X(\theta)Y(\omega - \theta)d\theta$
Differencing in Time	$x[n] - x[n-1]$	$(1 - e^{-j\omega})X(\omega)$
Accumulation	$\sum_{k=-\infty}^{\infty} x[k]$	$\frac{1}{1-e^{-j\omega}} + \pi X(0) \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$
Frequency Differentiation	$nx[n]$	$j\dfrac{dX(\omega)}{d\omega}$
Parseval's Relation for Aperiodic Signals	$\sum_{k=-\infty}^{\infty} \lvert x[k]\rvert^2$	$\frac{1}{2\pi}\int_{2\pi} \lvert X(\omega)\rvert^2 d\omega$

Table of Z-Transform Pairs:

$$\text{Z-Transform} \quad : \quad X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$$\text{Inverse Z-Transform} \quad : \quad x[n] = \frac{1}{2\pi j}\oint_{\mathcal{C}} X(z)z^{n-1}\,dz\ .$$

$x[n]$	$X(\omega)$	ROC
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
$\delta[n]$	1	All z
$\delta[n-n_0]$	z^{-n_0}	All z
$u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
$\cos(\omega_0 n)u[n]$	$\frac{1-z^{-1}\cos(\omega_0)}{1-2z^{-1}\cos(\omega_0)+z^{-2}}$	$\lvert z\rvert > 1$
$\sin(\omega_0 n)u[n]$	$\frac{z^{-1}\sin(\omega_0)}{1-2z^{-1}\cos(\omega_0)+z^{-2}}$	$\lvert z\rvert > 1$
$a^n\cos(\omega_0 n)u[n]$	$\frac{1-az^{-1}\cos(\omega_0)}{1-2az^{-1}\cos(\omega_0)+a^2z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$
$a^n\sin(\omega_0 n)u[n]$	$\frac{az^{-1}\sin(\omega_0)}{1-a2z^{-1}\cos(\omega_0)+a^2z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$

Table of Z-Transform Properties: For each property, assume

$$x[n] \overset{Z}{\longleftrightarrow} X(z) \quad \text{and} \quad y[n] \overset{Z}{\longleftrightarrow} Y(z)$$

Property	Time domain	Z-domain
Linearity	$Ax[n] + By[n]$	$AX(z) + BY(z)$
Time Shifting	$x[n - n_0]$	$X(z)z^{-n_0}$
Z-scaling	$a^n x[n]$	$X(a^{-1}z)$
Conjugation	$x^*[n]$	$X^*(z^*)$
Time Reversal	$x[-n]$	$X(z^{-1})$
Convolution	$x[n] * y[n]$	$X(z)Y(z)$
Differentiation in z-domain	$nx[n]$	$-z\frac{dX(z)}{dz}$
Initial Value Theorem	$x[n]$ is causal	$x(0) = \lim_{z\to\infty} X(z)$
Final Value Theorem	$x[n]$ is causal	$x(\infty) = \lim_{z\to 1}[z - 1]X(z)$