

Table of Fourier Series Properties:

$$\text{Fourier Analysis} : c_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-jk\Omega_0 t} dt$$

$$\text{Fourier Synthesis} : x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\Omega_0 t}$$

(Ω_0 is the *fundamental angular frequency* of $x(t)$ and T_0 is the *fundamental period* of $x(t)$)

For each property, assume $x(t) \xleftrightarrow{\mathcal{F}} c_k$ and $y(t) \xleftrightarrow{\mathcal{F}} d_k$

Property	Time domain	Fourier domain
Linearity	$Ax(t) + By(t)$	$Ac_k + Bd_k$
Time Shifting	$x(t - t_0)$	$c_k e^{-jk\Omega_0 t_0}$
Frequency Shifting	$x(t) e^{jM\Omega_0 t}$	c_{k-M}
Conjugation	$x^*(t)$	c_{-k}^*
Time Reversal	$x(-t)$	c_{-k}
Circular Conv.	$x(t) \otimes y(t)$	$T_0 c_k d_k$
Multiplication	$x(t)y(t)$	$c_k * d_k$
Differentiation	$\frac{d}{dt} x(t)$	$(jk\Omega_0) c_k$
Integration	$\int_{-\infty}^t x(\tau) d\tau$	$\left(\frac{1}{jk\Omega_0}\right) c_k$
Conjugate Symmetry for		
Real Signals	$x(t)$ is real	$c_k = c_{-k}^*$
Real and Even Signals	$x(t)$ is real and even	c_k is real and even
Real and Odd Signals	$x(t)$ is real and odd	c_k is purely imaginary and odd
Parseval's Relation for		
Cont. Periodic signals	$\frac{1}{T_0} \int_{T_0} x(t) ^2 dt$	$\sum_{k=-\infty}^{\infty} c_k ^2$

Table of Special Functions:

Function name	Expression	Notes
Sinc function	$\text{sinc}(x) = \frac{\sin(x)}{x}$	Note that there exists an alternative definition for $\text{sinc}(x)$
Rectangle function	$\text{rect}(x) = \begin{cases} 0 & \text{if } x \geq 1/2 \text{ , } x < -1/2 \\ 1 & \text{if } -1/2 \leq x < 1/2 \end{cases}$ $\text{rect}(x) = u(x + \frac{1}{2}) - u(x - \frac{1}{2})$	Note that there are alternative definitions with different values for $\text{rect}(\pm 1/2)$
Unit triangle function	$\Delta(x) = \begin{cases} 0 & \text{if } x \geq 1/2 \\ 1 - 2 x & \text{if } x < 1/2 \end{cases}$	

Table of Fourier Transform Pairs:

$$\text{Fourier Transform : } X(\Omega) = \int_{-\infty}^{\infty} x(t)e^{-j\Omega t} dt$$

$$\text{Inverse Fourier Transform : } x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\Omega)e^{j\Omega t} d\Omega .$$

$x(t)$	$X(\Omega)$	condition
$e^{-at}u(t)$	$\frac{1}{a+j\Omega}$	$a > 0$
$e^{at}u(-t)$	$\frac{1}{a-j\Omega}$	$a > 0$
$e^{-a t }$	$\frac{2a}{a^2+\Omega^2}$	$a > 0$
$te^{-at}u(t)$	$\frac{1}{(a+j\Omega)^2}$	$a > 0$
$t^n e^{-at}u(t)$	$\frac{n!}{(a+j\Omega)^{n+1}}$	$a > 0$
$\delta(t)$	1	
1	$2\pi\delta(\Omega)$	
$e^{j\Omega_0 t}$	$2\pi\delta(\Omega - \Omega_0)$	
$\cos(\Omega_0 t)$	$\pi (\delta(\Omega - \Omega_0) + \delta(\Omega + \Omega_0))$	
$\sin(\Omega_0 t)$	$j\pi (\delta(\Omega + \Omega_0) - \delta(\Omega - \Omega_0))$	
$u(t)$	$\pi\delta(\Omega) + \frac{1}{j\Omega}$	
$\text{sgn}(t)$	$\frac{2}{j\Omega}$	
$\cos(\Omega_0 t)u(t)$	$\frac{\pi}{2} (\delta(\Omega - \Omega_0) + \delta(\Omega + \Omega_0)) + \frac{j\Omega}{\Omega_0^2 - \Omega^2}$	
$\sin(\Omega_0 t)u(t)$	$\frac{\pi}{2j} (\delta(\Omega - \Omega_0) - \delta(\Omega + \Omega_0)) + \frac{\Omega_0}{\Omega_0^2 - \Omega^2}$	
$e^{-at} \sin(\Omega_0 t)u(t)$	$\frac{\Omega_0}{(a+j\Omega)^2 + \Omega_0^2}$	$a > 0$
$e^{-at} \cos(\Omega_0 t)u(t)$	$\frac{a+j\Omega}{(a+j\Omega)^2 + \Omega_0^2}$	$a > 0$
$\text{rect}\left(\frac{t}{T}\right)$	$T \text{sinc}\left(\frac{\Omega T}{2}\right)$	
$\frac{W}{\pi} \text{sinc}(Wt)$	$\text{rect}\left(\frac{\Omega}{2W}\right)$	
$\Delta\left(\frac{t}{T}\right)$	$\frac{T}{2} \text{sinc}^2\left(\frac{\Omega T}{4}\right)$	
$\frac{W}{2\pi} \text{sinc}^2\left(\frac{WT}{2}\right)$	$\Delta\left(\frac{\Omega}{2W}\right)$	
$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\Omega_0 \sum_{n=-\infty}^{\infty} \delta(\Omega - n\Omega_0)$	$\Omega_0 = \frac{2\pi}{T_0}$
$e^{-t^2/(2\sigma^2)}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\Omega^2/2}$	

Table of Fourier Transform Properties: For each property, assume

$$x(t) \xleftrightarrow{\mathcal{F}} X(\Omega) \quad \text{and} \quad y(t) \xleftrightarrow{\mathcal{F}} Y(\Omega)$$

Property	Time domain	Fourier domain
Linearity	$Ax(t) + By(t)$	$AX(\Omega) + BY(\Omega)$
Time Shifting	$x(t - t_0)$	$X(\Omega)e^{-j\Omega t_0}$
Time Scaling	$x(\alpha t)$	$\frac{1}{ \alpha }X\left(\frac{\Omega}{\alpha}\right)$
Frequency Shifting	$x(t)e^{j\Omega_0 t}$	$X(\Omega - \Omega_0)$
Conjugation	$x^*(t)$	$X^*(-\Omega)$
Time Reversal	$x(-t)$	$X(-\Omega)$
Convolution	$x(t) * y(t)$	$X(\Omega)Y(\Omega)$
Multiplication	$x(t)y(t)$	$\frac{1}{2\pi}X(\Omega) * Y(\Omega)$
Differentiation	$\frac{d}{dt}x(t)$	$(j\Omega)X(\Omega)$
Integration	$\int_{-\infty}^t x(\tau) d\tau$	$\left(\frac{1}{j\Omega}\right)X(\Omega)$
Conjugate Symmetry for		
Real Signals	$x(t)$ is real	$X(\Omega) = X^*(-\Omega)$
Real and Even Signals	$x(t)$ is real and even	$X(\Omega)$ is real and even
Real and Odd Signals	$x(t)$ is real and odd	$X(\Omega)$ is purely imaginary and odd
Parseval's Relation for		
Aperiodic signals	$\int_{-\infty}^{\infty} x(t) ^2 dt$	$\frac{1}{2\pi} \int_{-\infty}^{\infty} X(\Omega) ^2 d\Omega$

Table of Laplace Transform Pairs:

$$\begin{aligned} \text{Bilateral Laplace Transform} & : X(s) = \int_{-\infty}^{\infty} x(t)e^{-st} dt \\ \text{Unilateral Laplace Transform} & : X(s) = \int_0^{\infty} x(t)e^{-st} dt \\ \text{Inverse Laplace Transform} & : x(t) = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} X(s)e^{st} ds . \end{aligned}$$

$x(t)$	$X(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
$tu(t)$	$\frac{1}{s^2}$
$t^n u(t)$	$\frac{n!}{s^{n+1}}$
$e^{at}u(t)$	$\frac{1}{s-a}$
$te^{at}u(t)$	$\frac{1}{(s-a)^2}$
$t^n e^{at}u(t)$	$\frac{n!}{(s-a)^{n+1}}$
$\cos(bt)u(t)$	$\frac{s}{s^2 + b^2}$
$\sin(bt)u(t)$	$\frac{b}{s^2 + b^2}$
$e^{-at} \cos(bt)u(t)$	$\frac{s+a}{(s+a)^2 + b^2}$
$e^{-at} \sin(bt)u(t)$	$\frac{b}{(s+a)^2 + b^2}$
$re^{-at} \cos(bt + \theta)u(t)$	$\frac{(r \cos(\theta))s + (ar \cos(\theta) - br \sin(\theta))}{s^2 + 2as + (a^2 + b^2)}$
$re^{-at} \cos(bt + \theta)u(t)$	$\frac{0.5re^{j\theta}}{s+a-jb} + \frac{0.5re^{-j\theta}}{s+a+jb}$
$re^{-at} \cos(bt + \theta)u(t)$	$\frac{As+B}{s^2 + 2as + c}$
$\theta = \tan^{-1} \left(\frac{Aa-B}{A\sqrt{c-a^2}} \right)$	$b = \sqrt{c-a^2}$
$e^{-at} \left[A\cos(bt) + \frac{B-Aa}{b} \sin(bt) \right] u(t)$	$b = \sqrt{c-a^2}$ $\frac{As+B}{s^2 + 2as + c}$

Table of Laplace Transform Properties: For each property, assume

$$x(t) \xleftrightarrow{\mathcal{L}} X(s) \quad \text{and} \quad y(t) \xleftrightarrow{\mathcal{L}} Y(s)$$

Property	Time domain	Laplace domain
Linearity	$Ax(t) + By(t)$	$AX(s) + BY(s)$
Time Shifting	$x(t - t_0)$	$X(s)e^{-st_0}$
Time Scaling	$x(\alpha t), \quad \alpha \geq 0$	$\frac{1}{\alpha} X\left(\frac{s}{\alpha}\right)$
Frequency Shifting	$x(t)e^{s_0 t}$	$X(s - s_0)$
Time Reversal	$x(-t)$	$X(-s)$
Convolution	$x(t) * y(t)$	$X(s)Y(s)$
Multiplication	$x(t)y(t)$	$\frac{1}{2\pi j} X(s) * Y(s)$
Differentiation	$\frac{d}{dt}x(t)$	$sX(s) - x(0^-)$
	$\frac{d^2}{dt^2}x(t)$	$s^2X(s) - sx(0^-) - \dot{x}(0^-)$
	$\frac{d^3}{dt^3}x(t)$	$s^3X(s) - s^2x(0^-) - s\dot{x}(0^-) - \ddot{x}(0^-)$
Integration	$\int_{0^-}^t x(\tau) d\tau$	$\left(\frac{1}{s}\right) X(s)$
	$\int_{-\infty}^t x(\tau) d\tau$	$\left(\frac{1}{s}\right) X(s) + \frac{1}{s} \int_{-\infty}^{0^-} x(t) dt$
Frequency Differentiation	$-tx(t)$	$\frac{dX(s)}{ds}$
Frequency Integration	$\frac{x(t)}{t}$	$\int_s^{\infty} X(z) dz$
Initial value	$x(0^+)$	$\lim_{s \rightarrow \infty} sX(s)$
Final value	$x(\infty)$	$\lim_{s \rightarrow 0} sX(s)$ (if poles of $sX(s)$ are in left-hand plane)