Lecture 24: Downsampling & Upsampling

Foundations of Digital Signal Processing

Outline

- Downsampling
- Downsampling with Anti-Aliasing Filter
- Upsampling
- Upsampling with Reconstruction Filter
- Resampling

News

- **Homework Assignment #8**
  - Due *Friday* by 11:59 PM
- **Coding Assignment #4**
  - Due *Today* by 11:59 PM
  - Submit via canvas
    - Submit answers as a PDF
    - Submit code as .m files
- **Exam #2**
  - In one week

Lecture 24: Downsampling & Upsampling

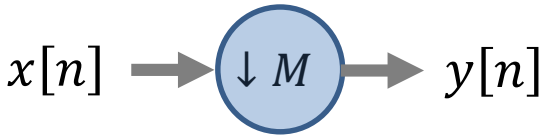
Foundations of Digital Signal Processing

Outline

- **Downsampling**
- Downsampling with Anti-Aliasing Filter
- Upsampling
- Upsampling with Reconstruction Filter
- Resampling

Downsampling

- **Reduce Sampling Rates**



Downsample by 2

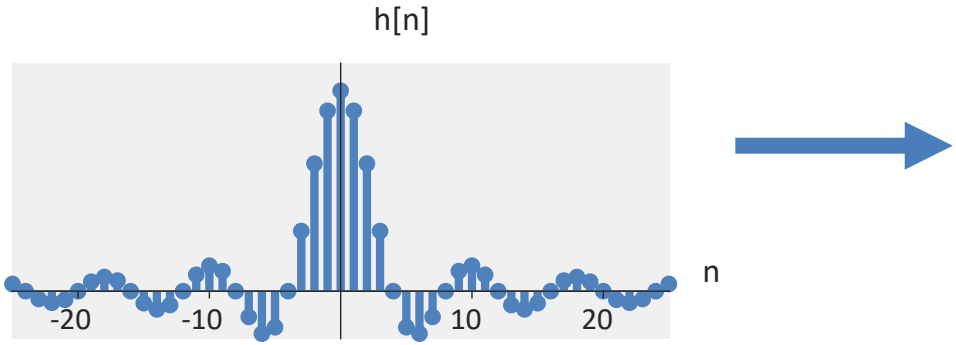
Image from Martin Vertelli's notes

Downsampling

- **Reduce Sampling Rates**

$$x[n] \rightarrow \boxed{\downarrow M} \rightarrow y[n] = x[Mn]$$

Downsample by M

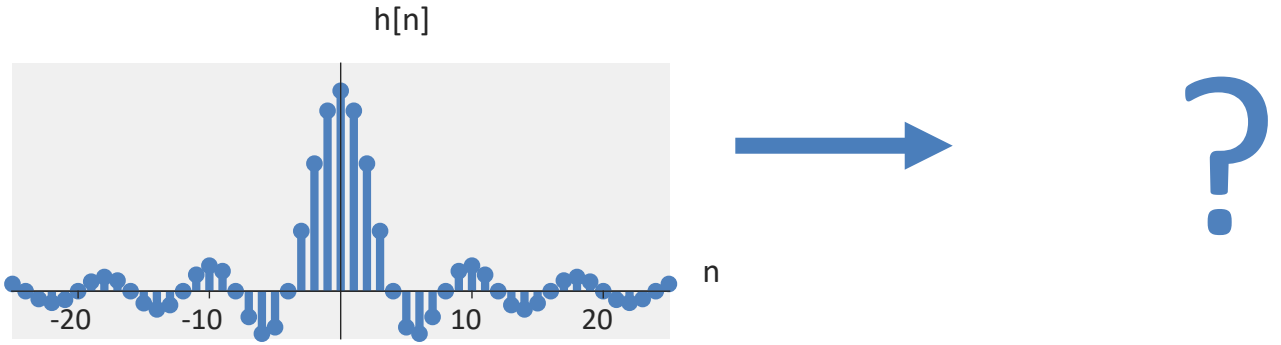


Downsampling

- **Reduce Sampling Rates**

$$x[n] \rightarrow \boxed{\downarrow 2} \rightarrow y[n] = x[2n]$$

Downsample by 2



Downsampling

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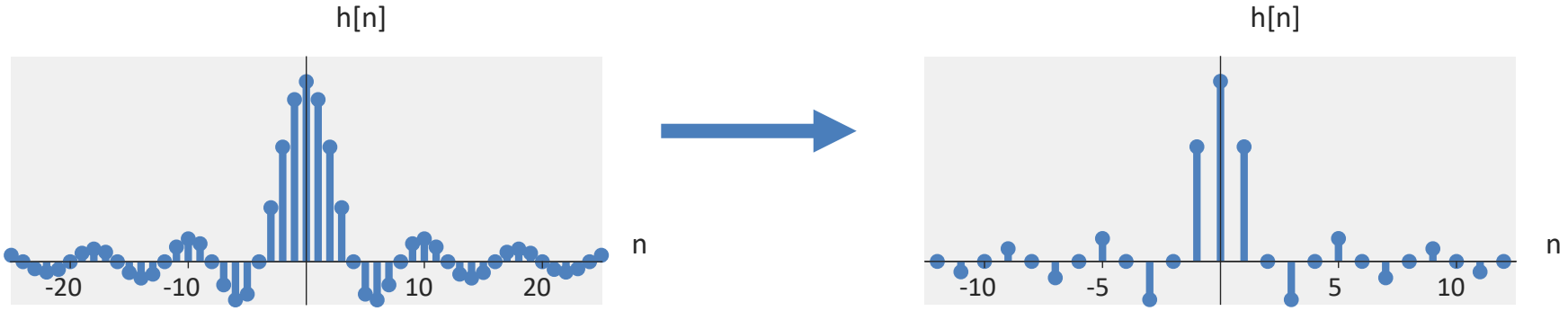

Downsampling

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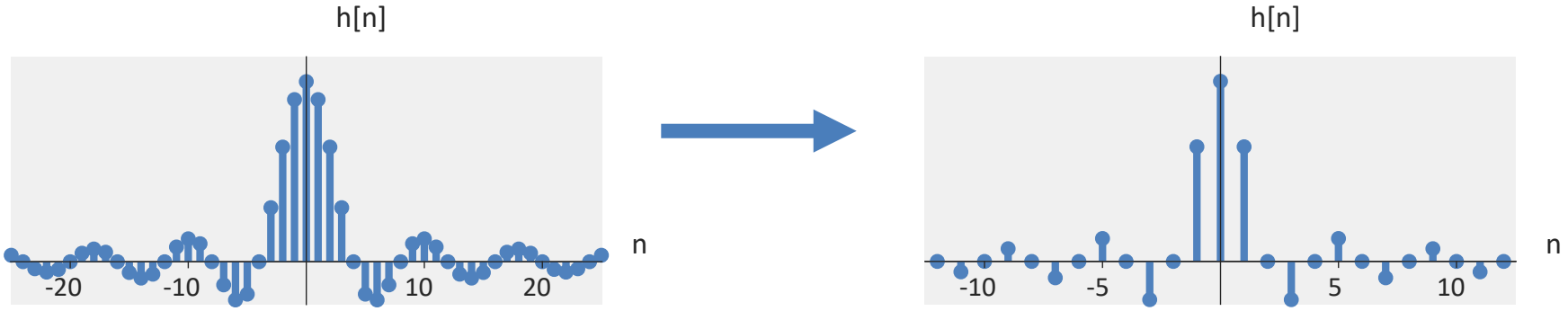
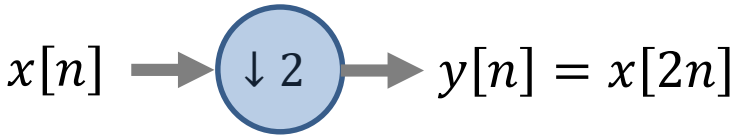


Downsampling

- **So what happens in frequency?**



  - Sketch each signal period by 2 AROUND every 2π
  - Reduce amplitude by 2 times

$$\mathcal{F}\{x[2n]\} = \frac{1}{2}[\mathrm{X}(\omega/2) + \mathrm{X}(\omega/2 - \pi)]$$

Downsampling

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  - Sketch each signal period by 2 AROUND every 2π
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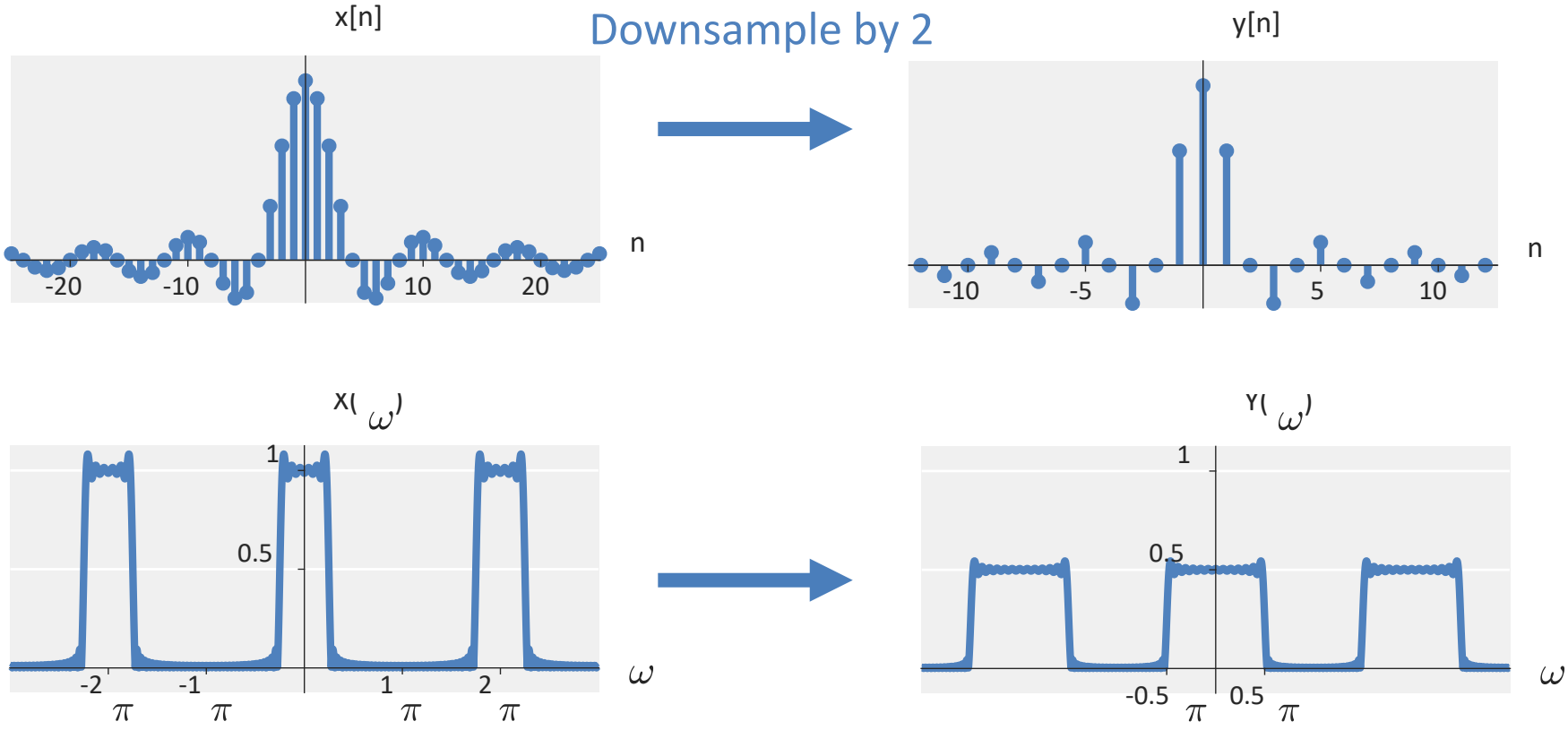

Downsampling

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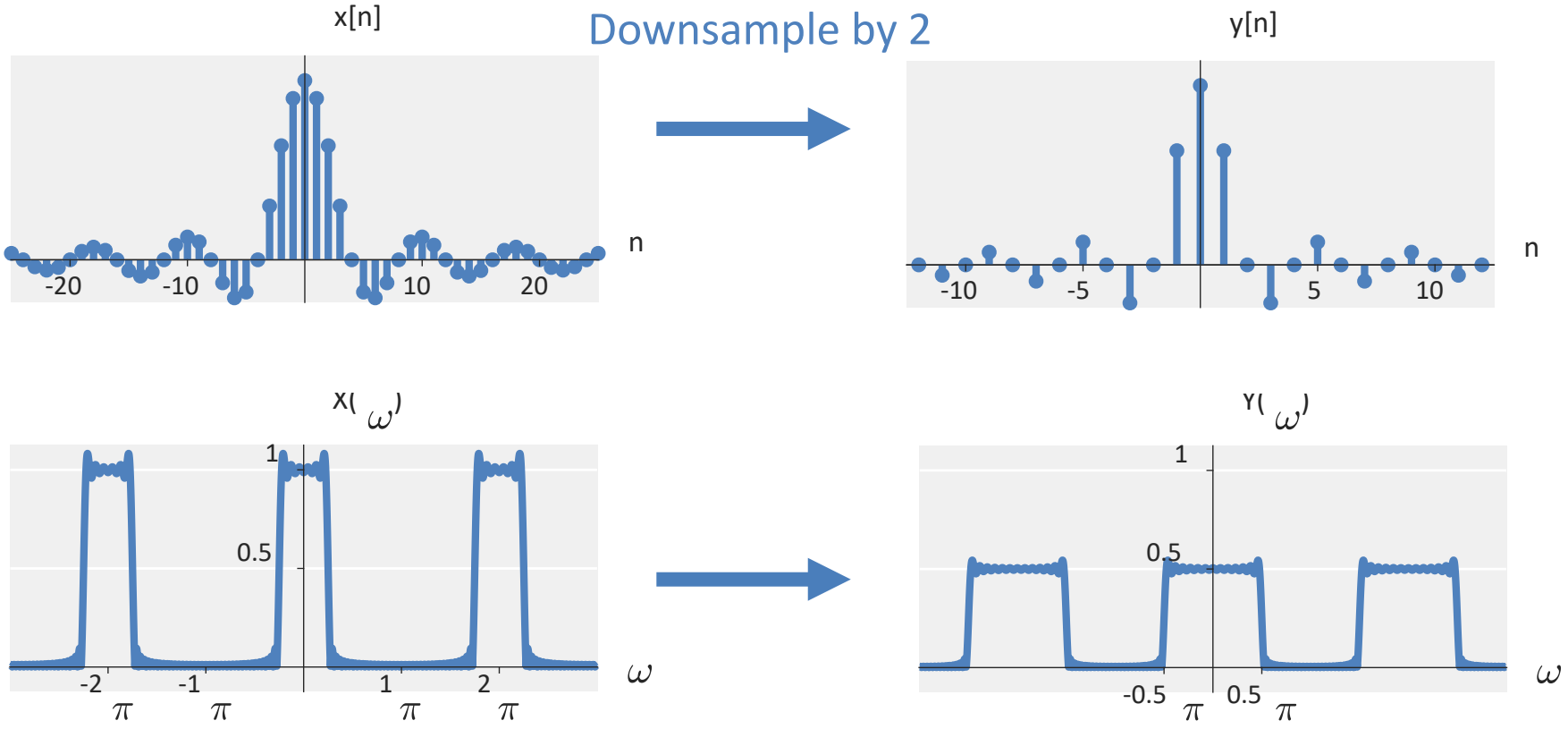

Downsampling

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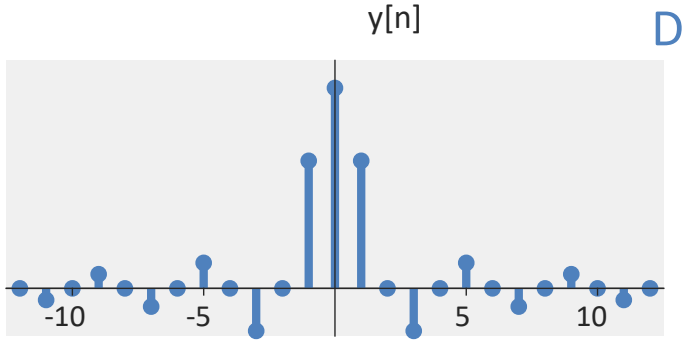
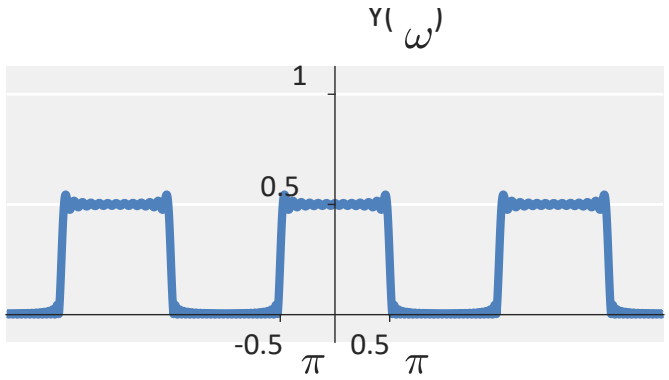


Downsample by 2

?



?

Downsampling

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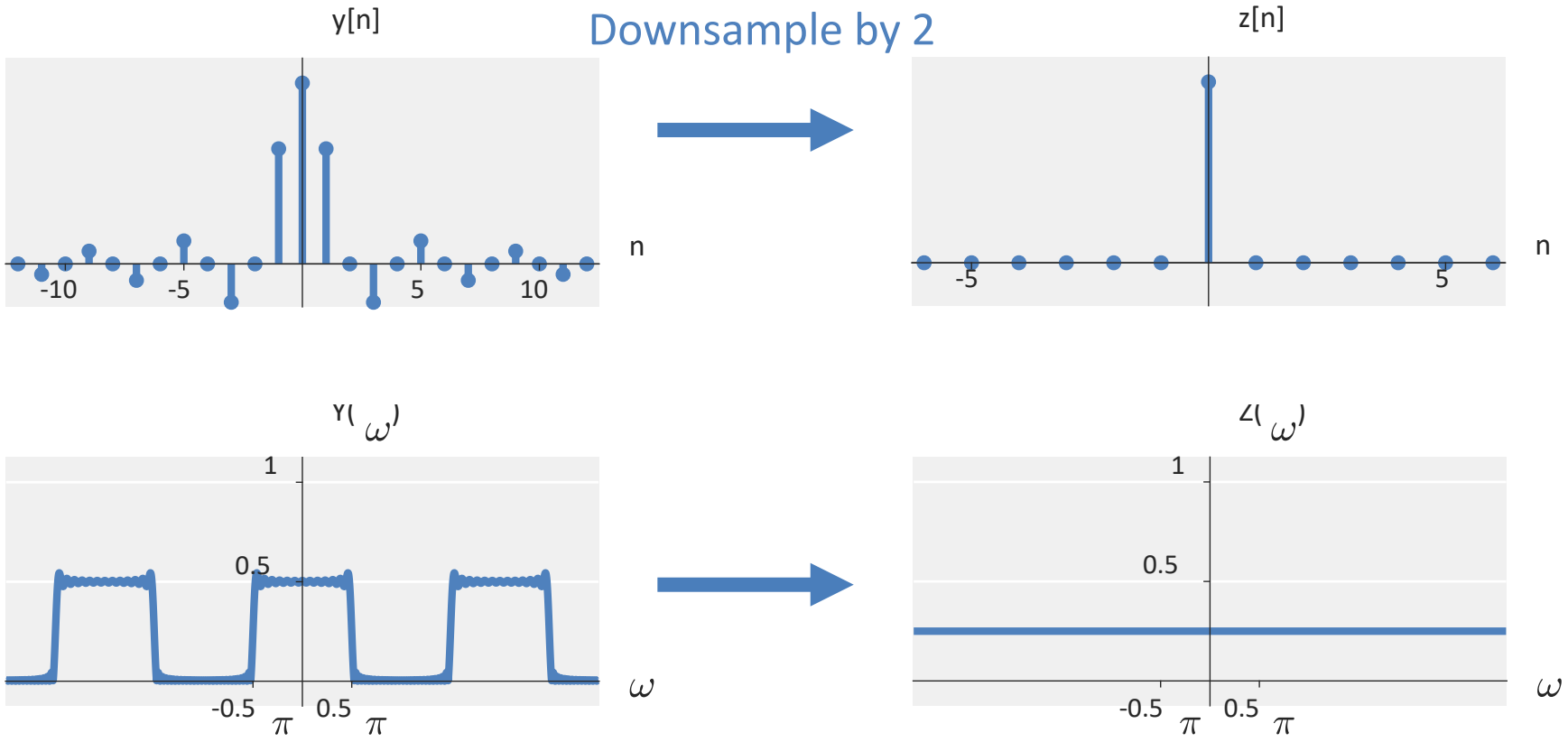

Downsampling

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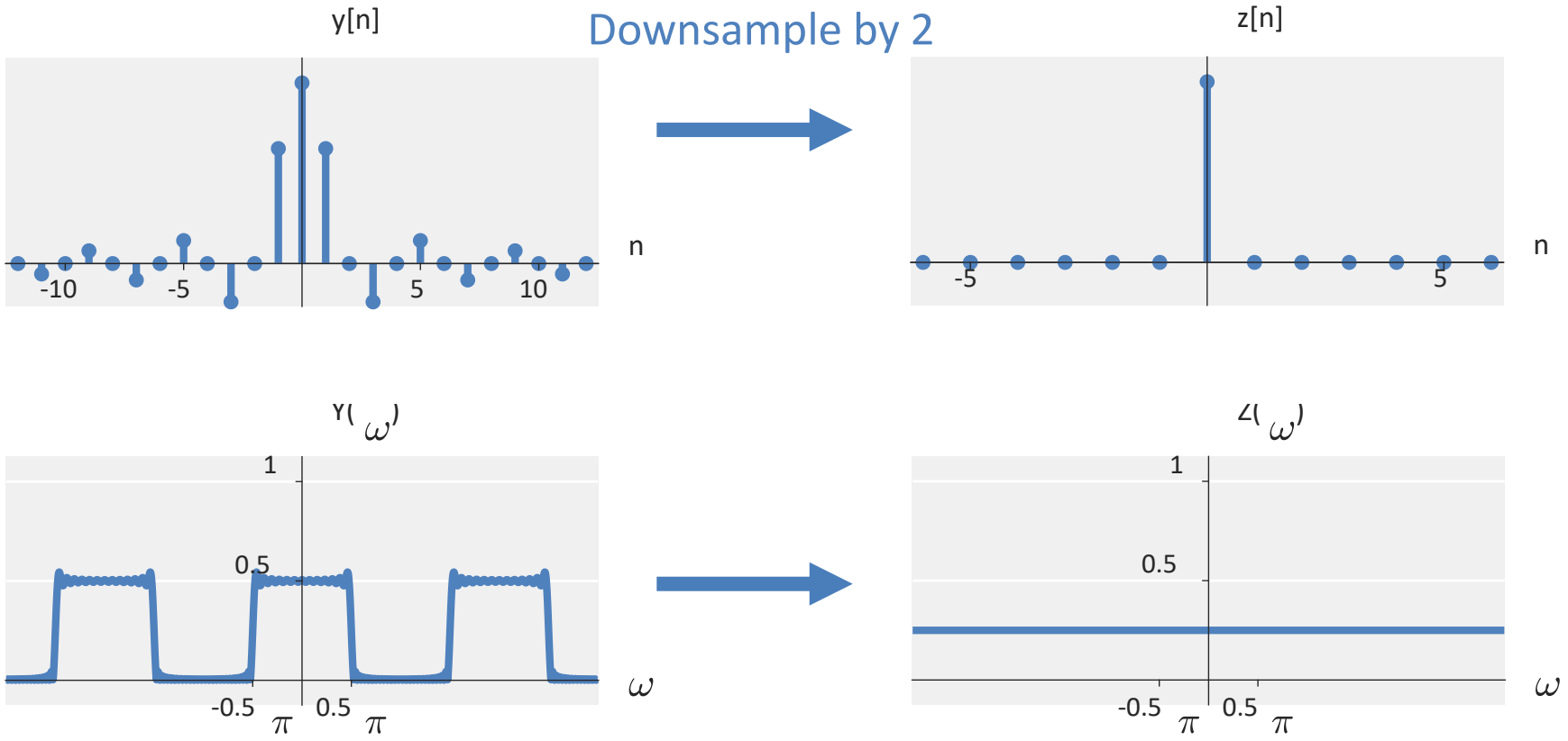

Downsampling

- **So what happens in frequency?**
  - Sketch each signal period by 2 AROUND every 2π
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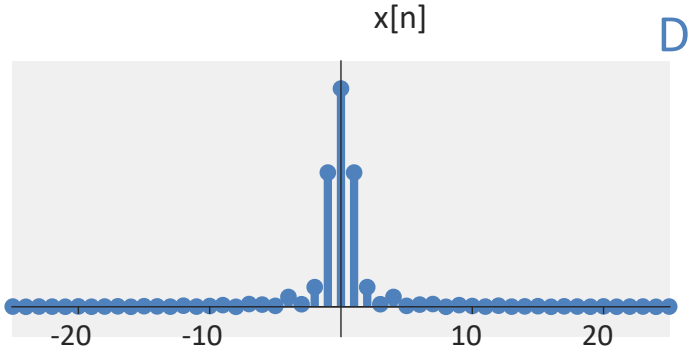
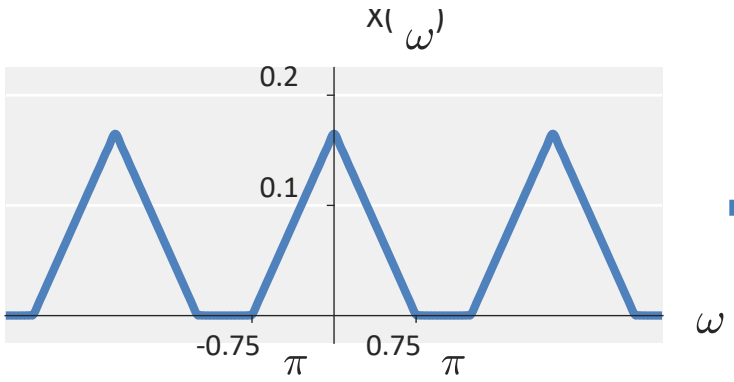


Downsample by 2

?



?

Downsampling

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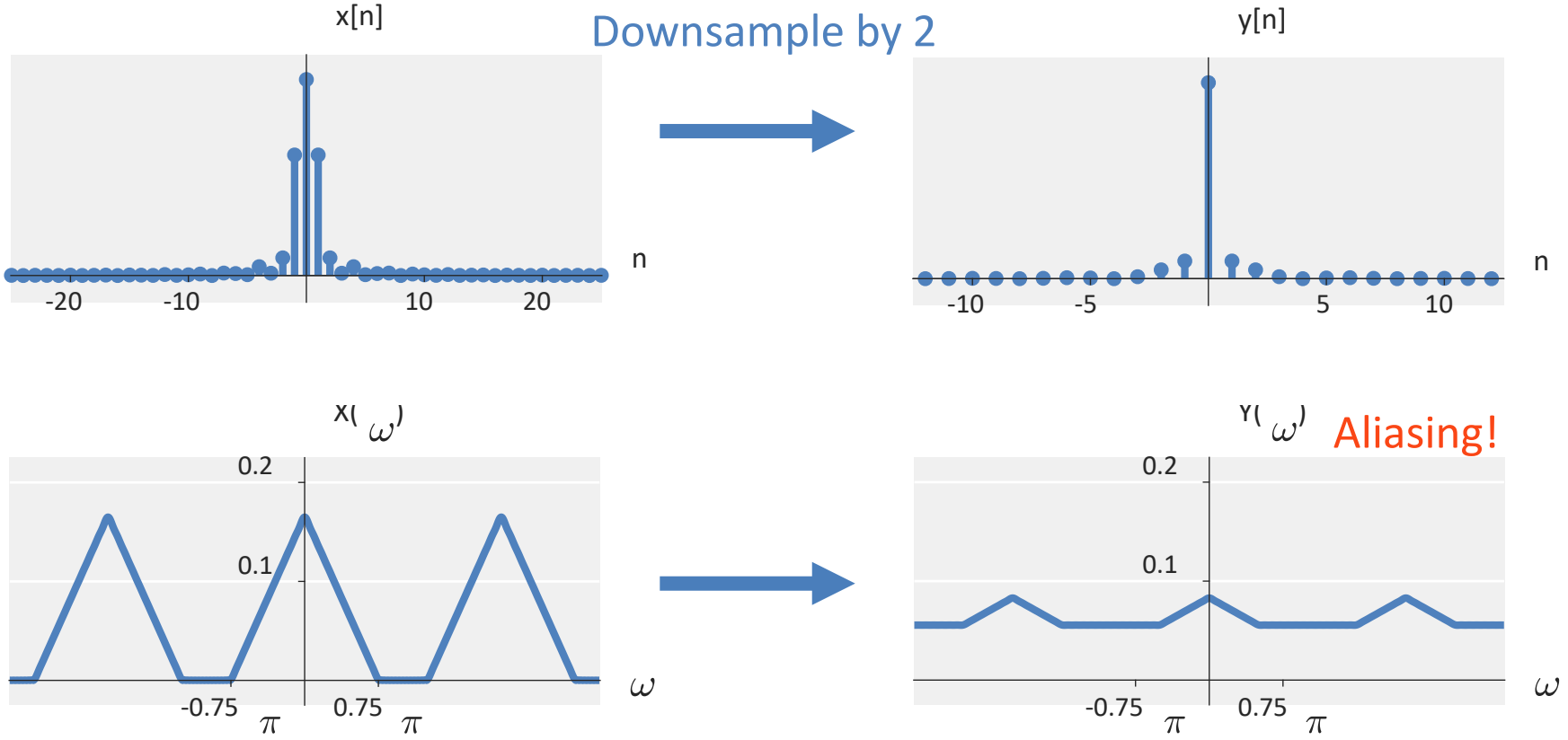
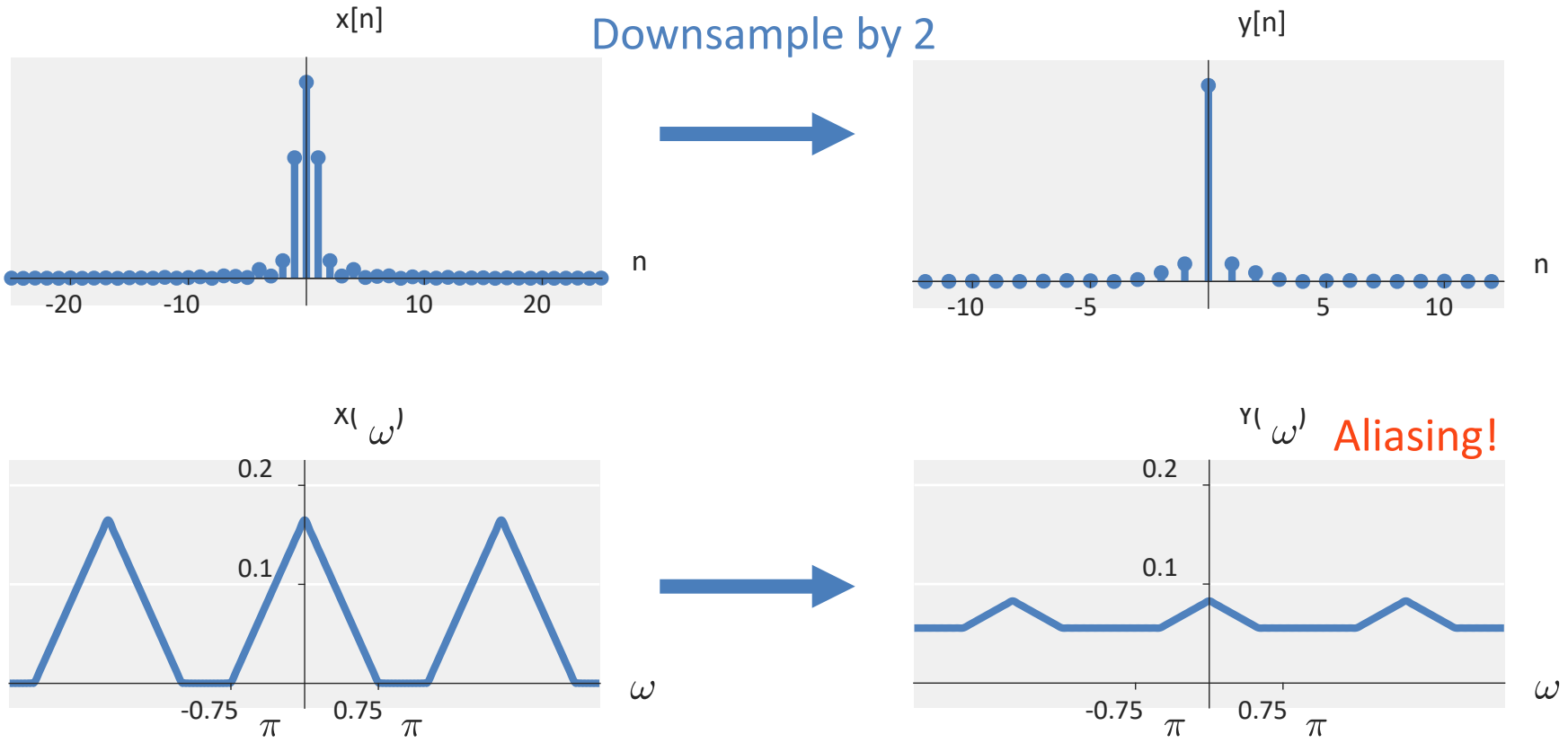


Downsampling

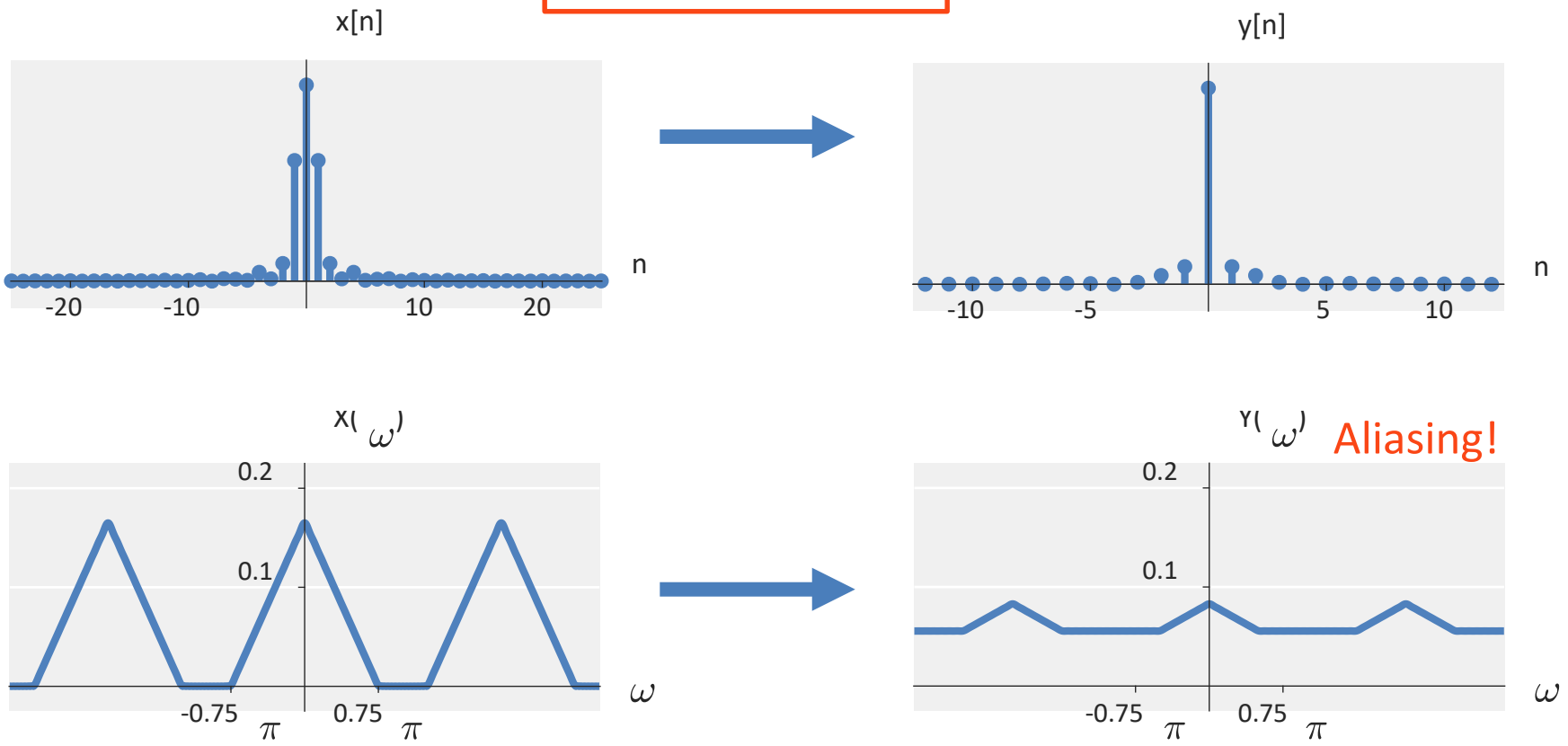
- **Question**
  - What is the condition to prevent aliasing for downsampling by 2?



Downsampling

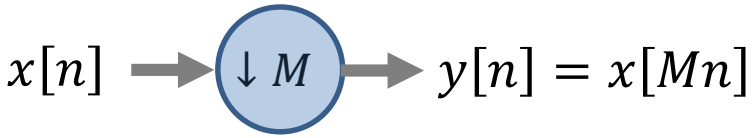
- **Question**
  - What is the condition to prevent aliasing for downsampling by 2?

$$\omega_{\max} < \pi/2$$



Downsampling

- **So what happens in frequency?**

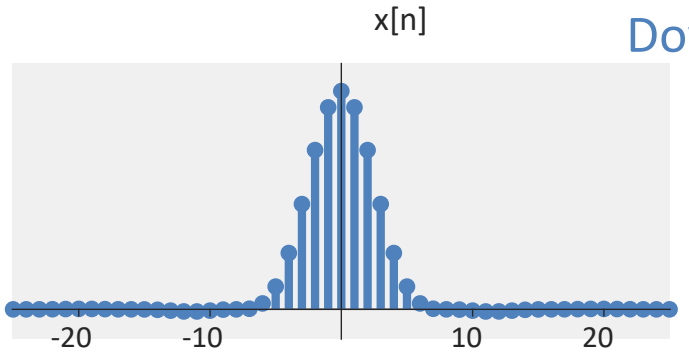


  - Sketch each signal period by M AROUND every 2π
  - Reduce amplitude by M times

$$\mathcal{F}\{x[Mn]\} = \frac{1}{M}\sum_{k=0}^{M-1} \mathrm{X}\left(\frac{\omega}{M} - \frac{2\pi}{M}k\right)$$

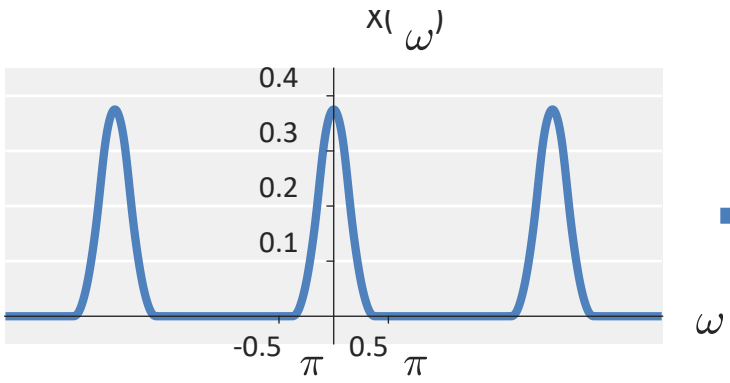
Downsampling

- **So what happens in frequency?**
  - Sketch each signal period by M AROUND every 2π
  - Reduce amplitude by M times



Downsample by M=4

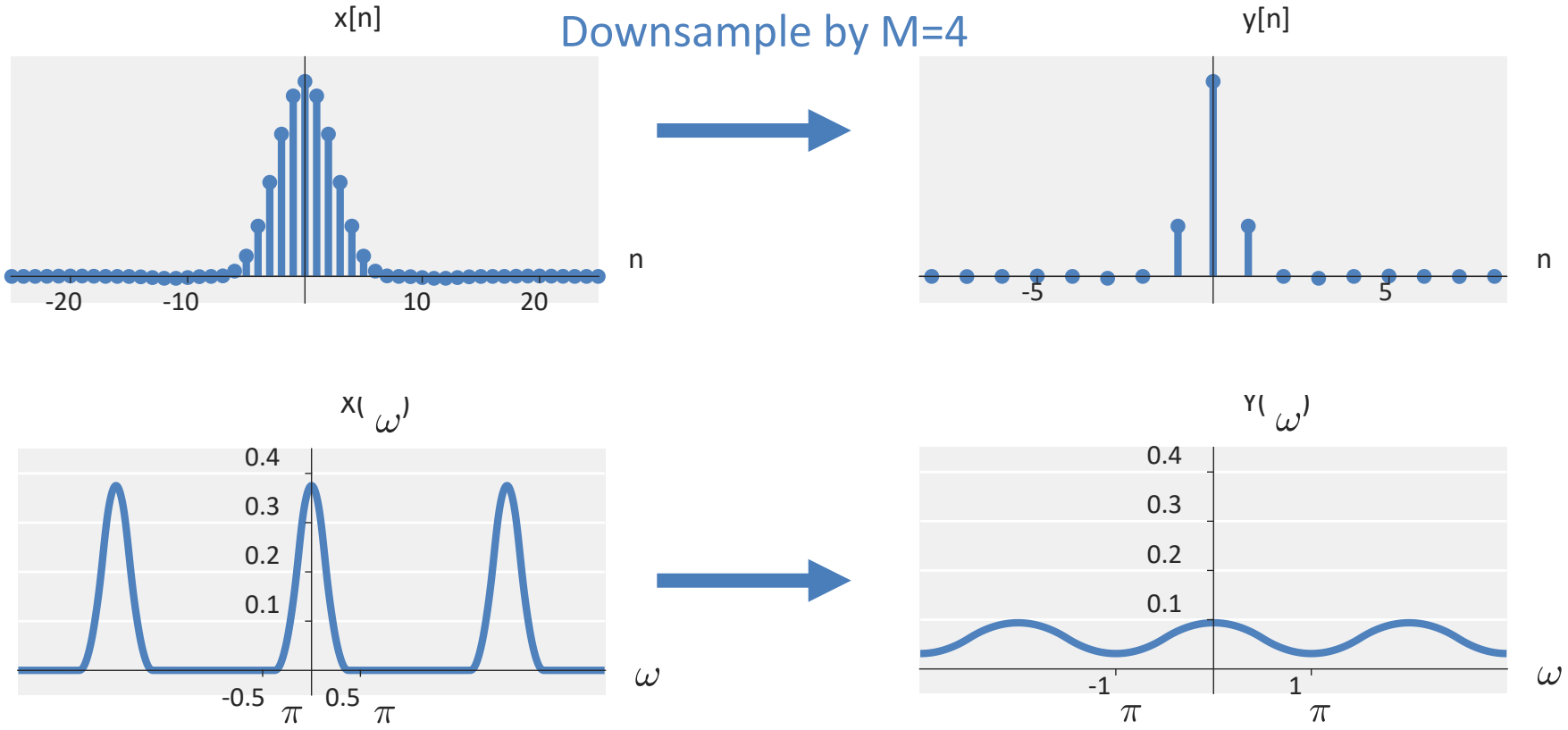
?



?

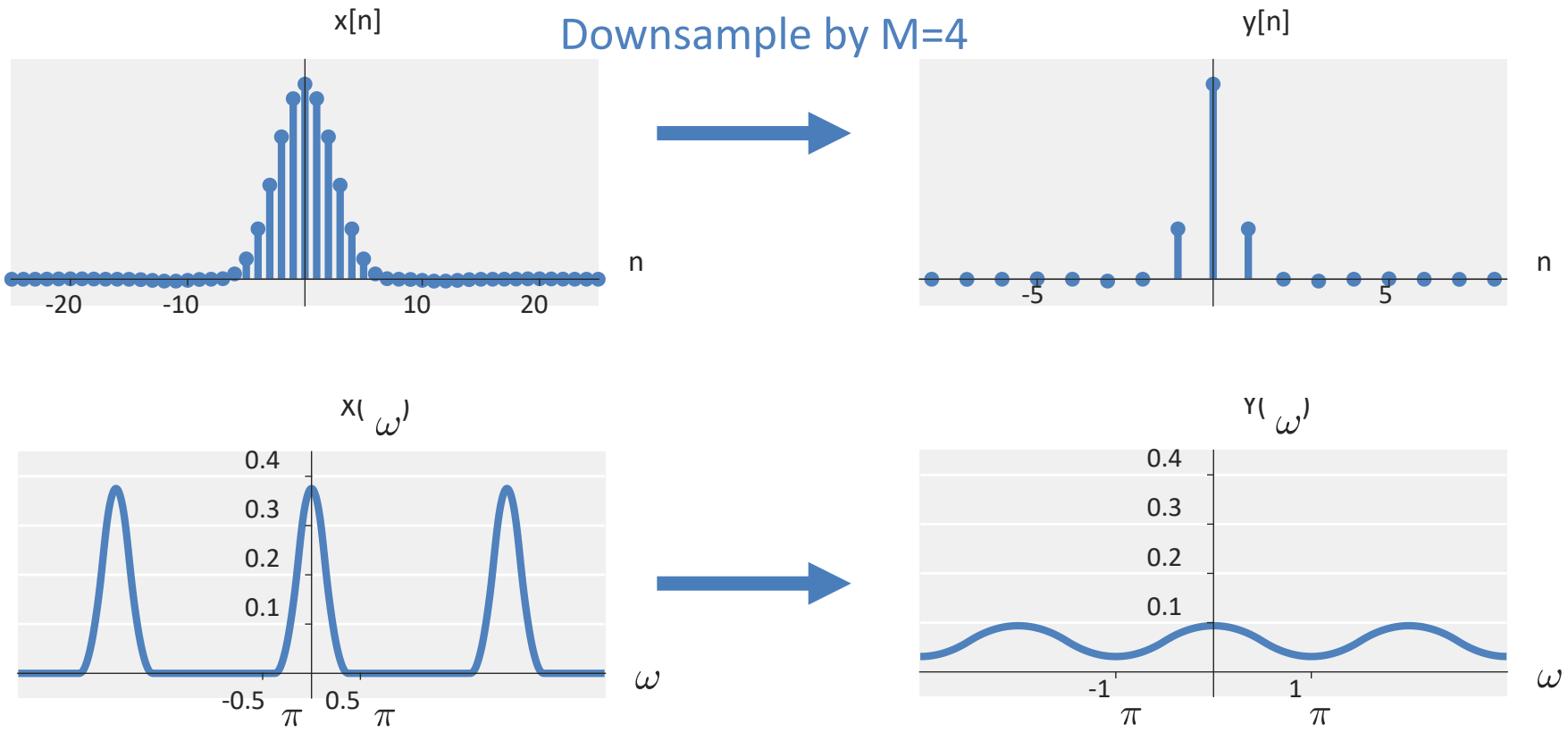
Downsampling

- **So what happens in frequency?**
  - Sketch each signal period by M AROUND every 2π
  - Reduce amplitude by M times



Downsampling

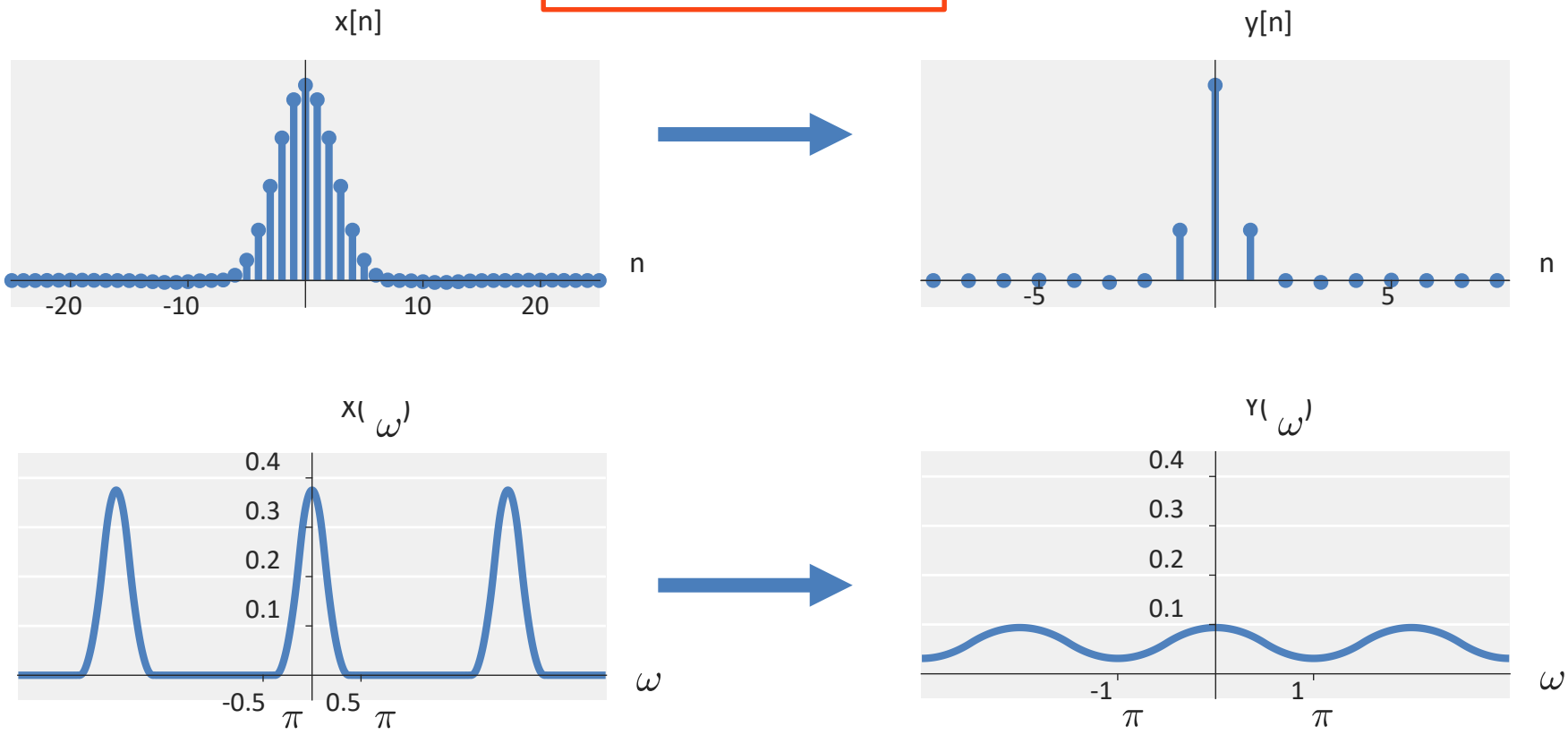
- **Question**
  - What is the condition to prevent aliasing for downsampling by 2?



Downsampling

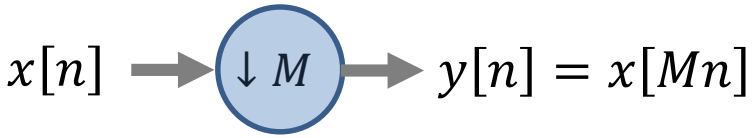
- **Question**
  - What is the condition to prevent aliasing for downsampling by 2?

$$\omega_{\max} < \pi / M$$



Downsampling

- **Question: How do we prevent aliasing?**

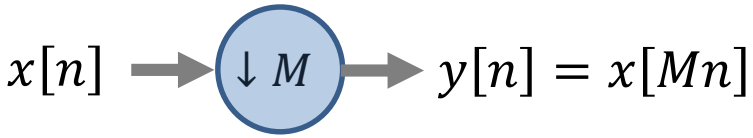


- Sketch each signal period by M AROUND every 2π
- Reduce amplitude by M times

$$\mathcal{F}\{x[Mn]\} = \frac{1}{M}\sum_{k=0}^{M-1} \mathrm{X}\left(\frac{\omega}{M} - \frac{2\pi}{M}k\right)$$

Downsampling

- **Question: How do we prevent aliasing?**

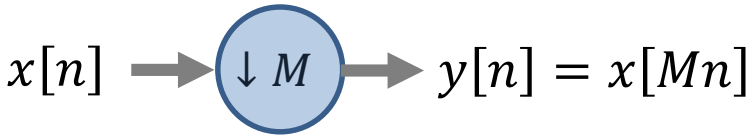


  - Sketch each signal period by M AROUND every 2π
  - Reduce amplitude by M times

- **Question:**
  - What is the requirement to prevent aliasing?

Downsampling

- **Question: How do we prevent aliasing?**



  - Sketch each signal period by M AROUND every 2π
  - Reduce amplitude by M times

- **Question:**
  - What is the requirement to prevent aliasing?
  - $M < \pi/M$

Lecture 24: Downsampling & Upsampling

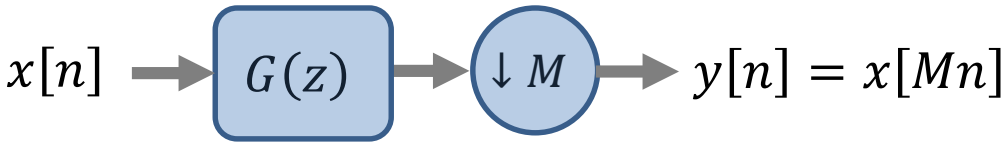
Foundations of Digital Signal Processing

Outline

- Downsampling
- **Downsampling with Anti-Aliasing Filter**
- Upsampling
- Upsampling with Reconstruction Filter
- Resampling

Downsampling

- **Question: How do we prevent aliasing?**



Low pass filter

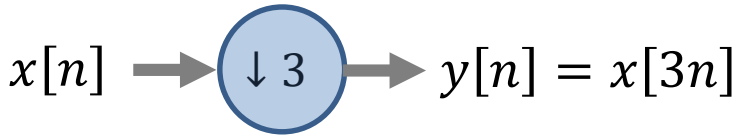
Cutoff: π/M

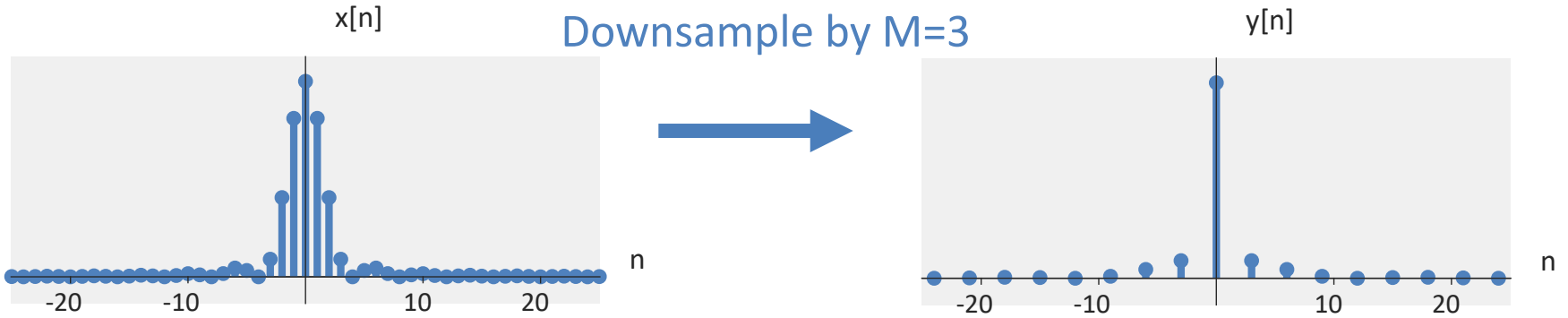
Gain: 1

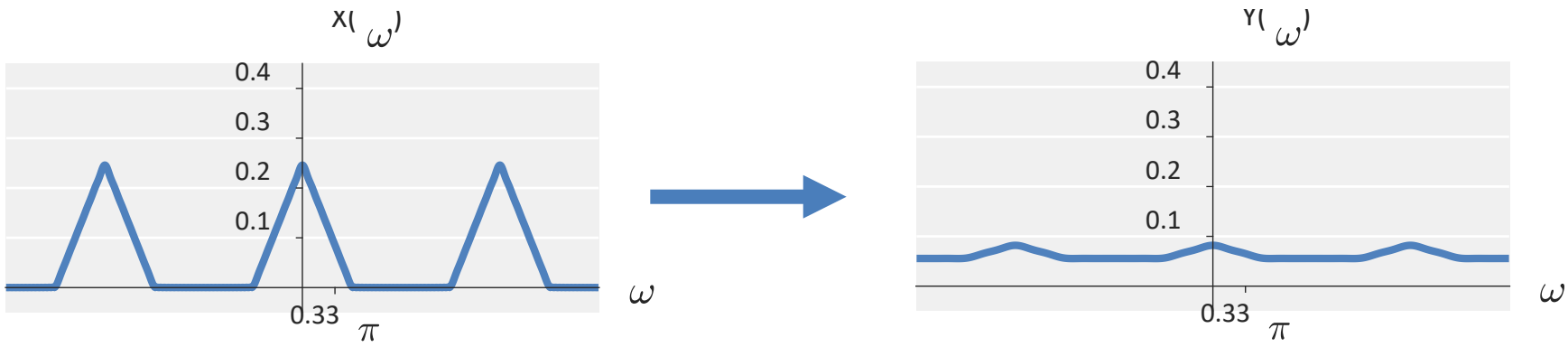
- Apply anti-aliasing
- Sketch each signal period by M AROUND every 2π
- Reduce amplitude by M times

Downsampling

- **Without Anti-Aliasing**

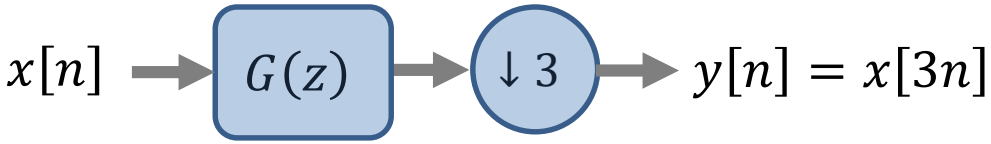


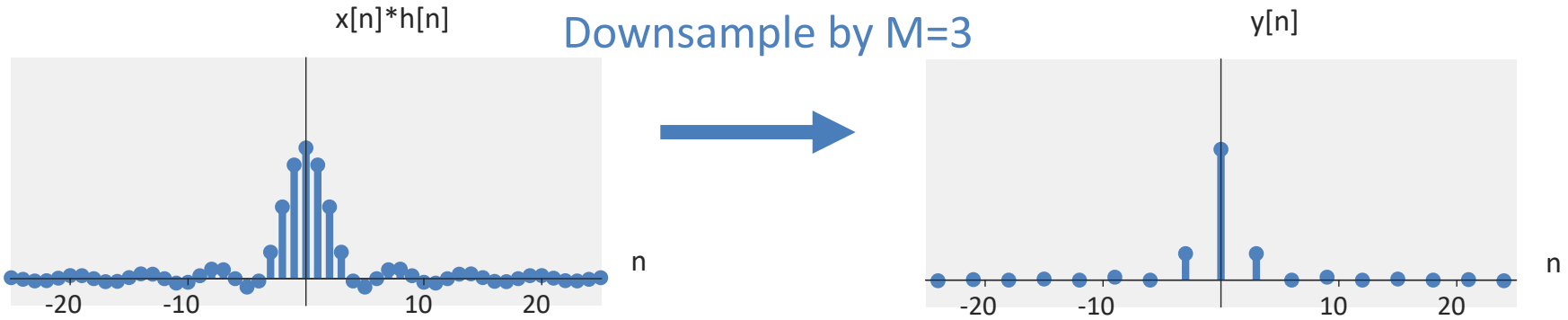




Downsampling

- **With Anti-Aliasing**





Lecture 24: Downsampling & Upsampling

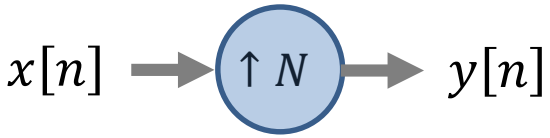
Foundations of Digital Signal Processing

Outline

- Downsampling
- Downsampling with Anti-Aliasing Filter
- **Upsampling**
- Upsampling with Reconstruction Filter
- Resampling

Upsampling

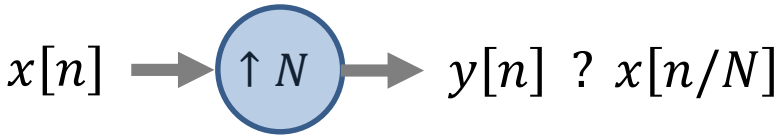
- **Increase Sampling Rates**



  - How do we increase the sampling rate?

Upsampling

- **Increase Sampling Rates**



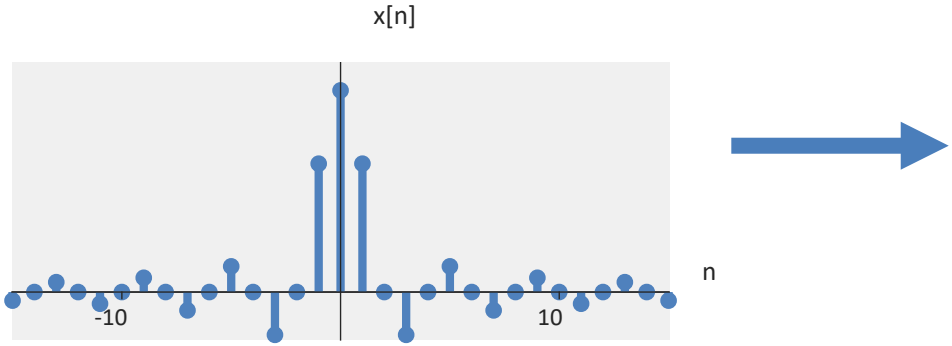
  - How do we increase the sampling rate?

Upsampling

- **Increase Sampling Rates**

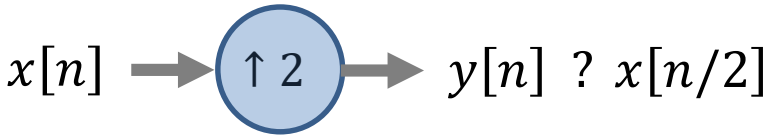
$$x[n] \rightarrow \boxed{\uparrow N} \rightarrow y[n] \;?\; x[n/N]$$

Upsample by M

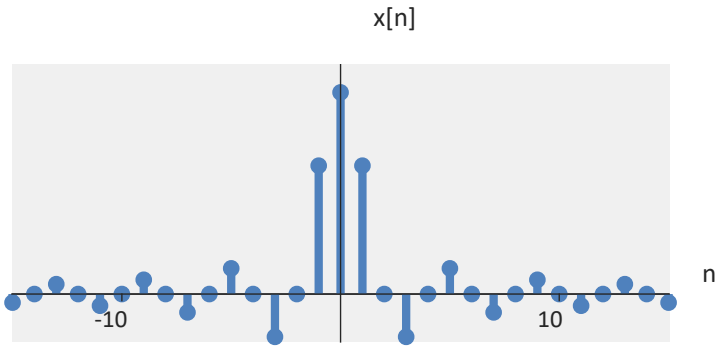


Upsampling

- **Increase Sampling Rates**

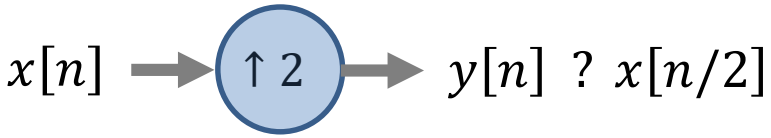


Upsample by 2

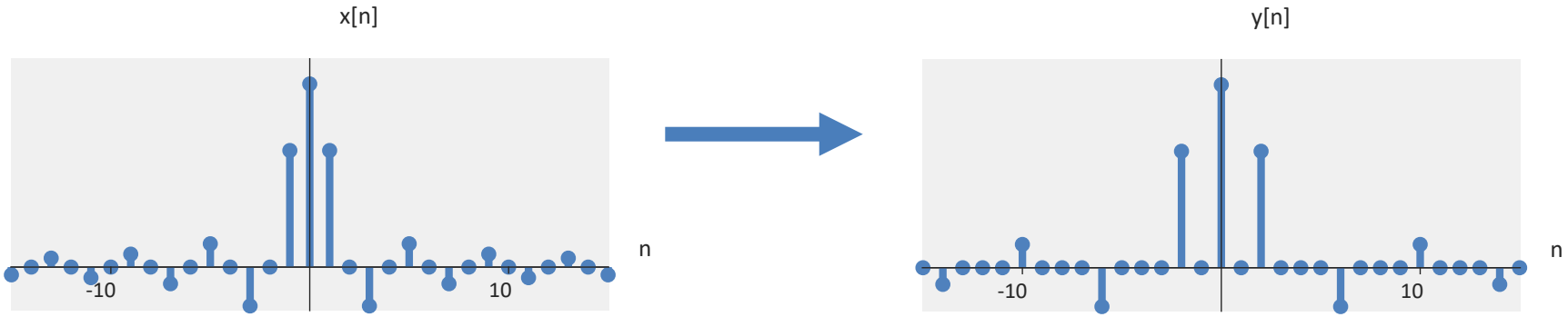


Upsampling

- **Increase Sampling Rates**

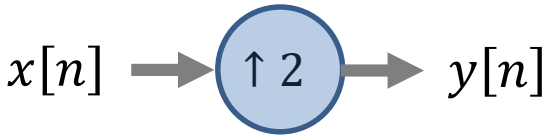


Upsample by 2



Upsampling

- **So what happens in frequency?**

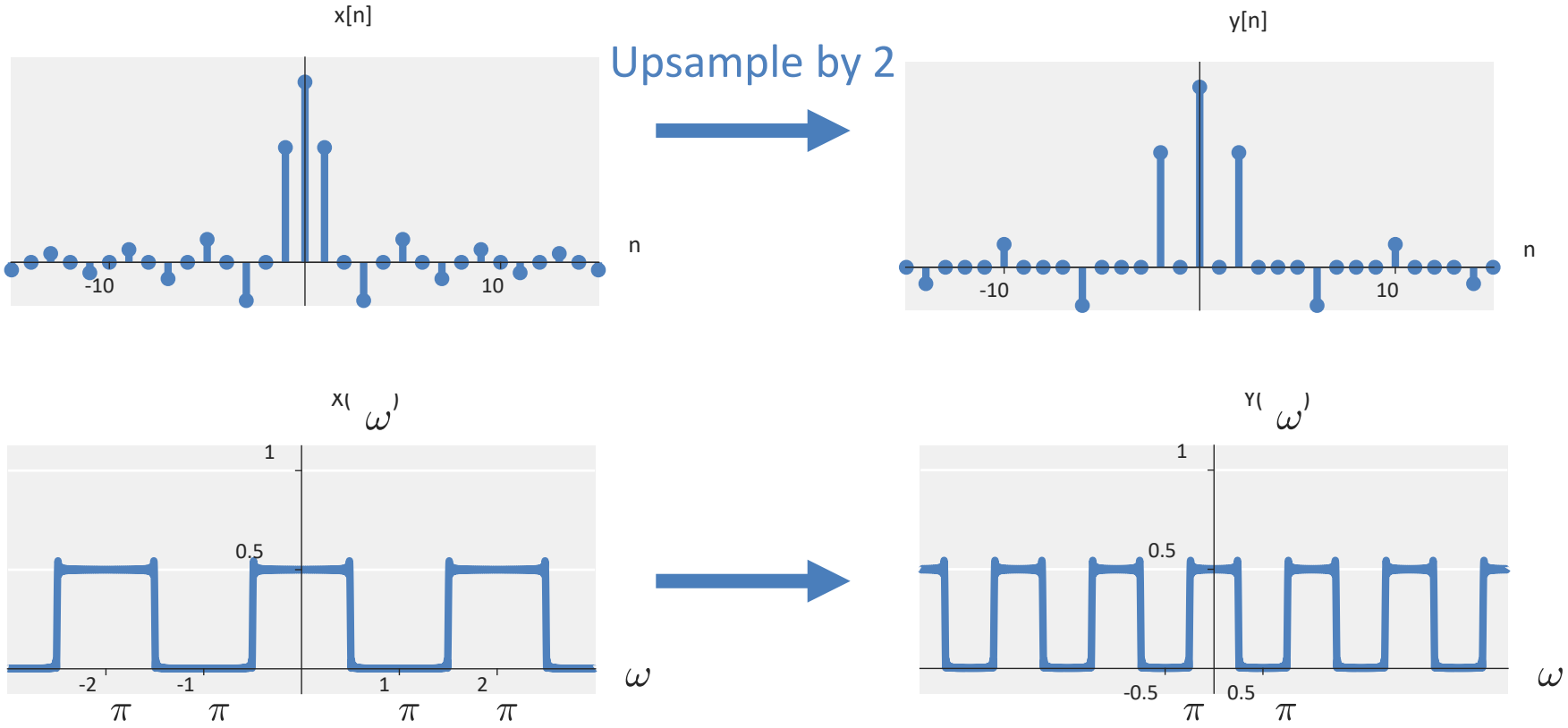


  - Shrink entire frequency-domain signal by 2
  - Amplitude does not change

$$\mathcal{F}\{y[n]\} = \mathrm{X}(2\omega)$$

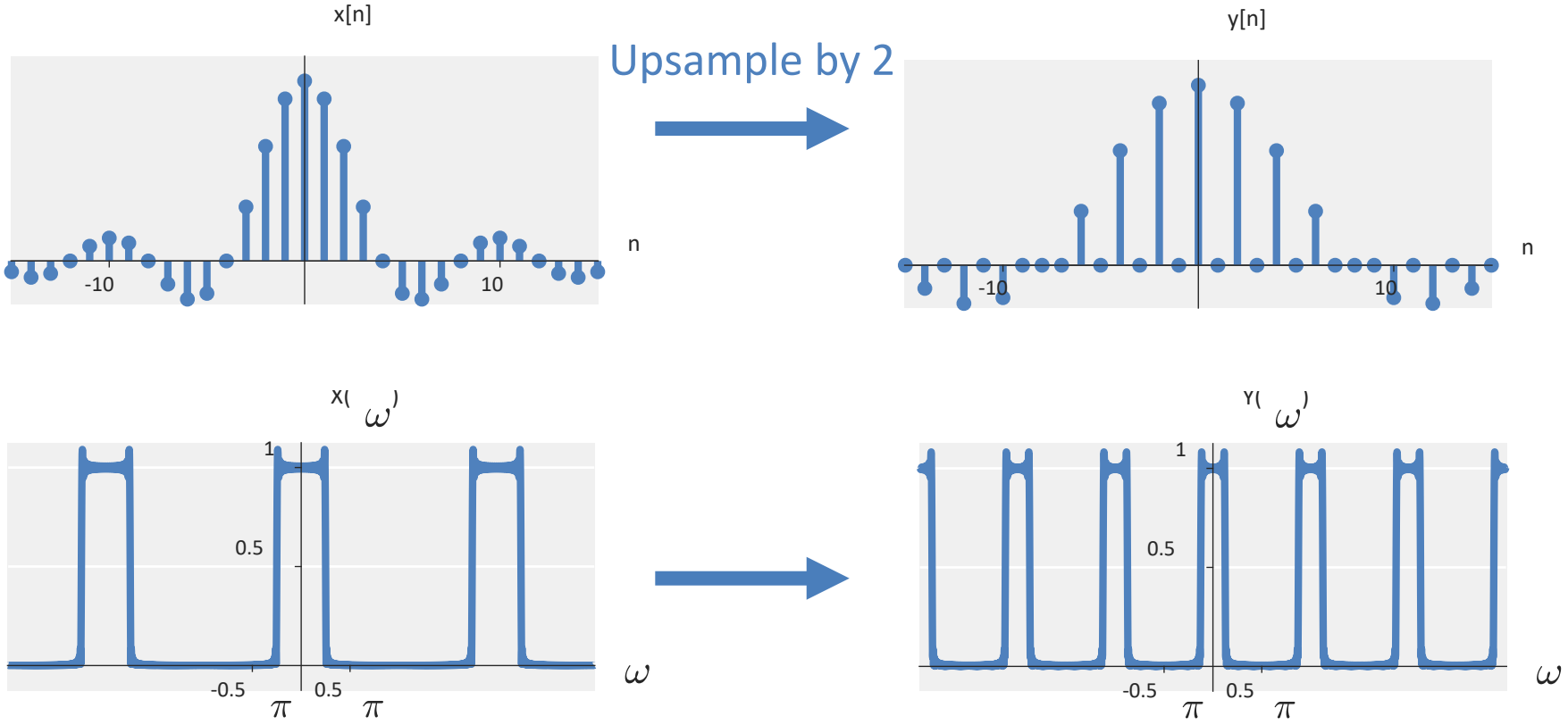
Upsampling

- **So what happens in frequency?**
  - Shrink entire frequency-domain signal by 2
  - Amplitude does not change



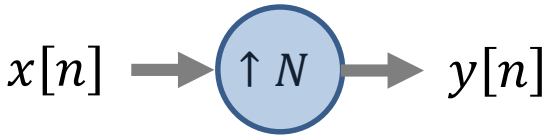
Upsampling

- **So what happens in frequency?**
  - Shrink entire frequency-domain signal by 2
  - Amplitude does not change



Upsampling

- **So what happens in frequency?**

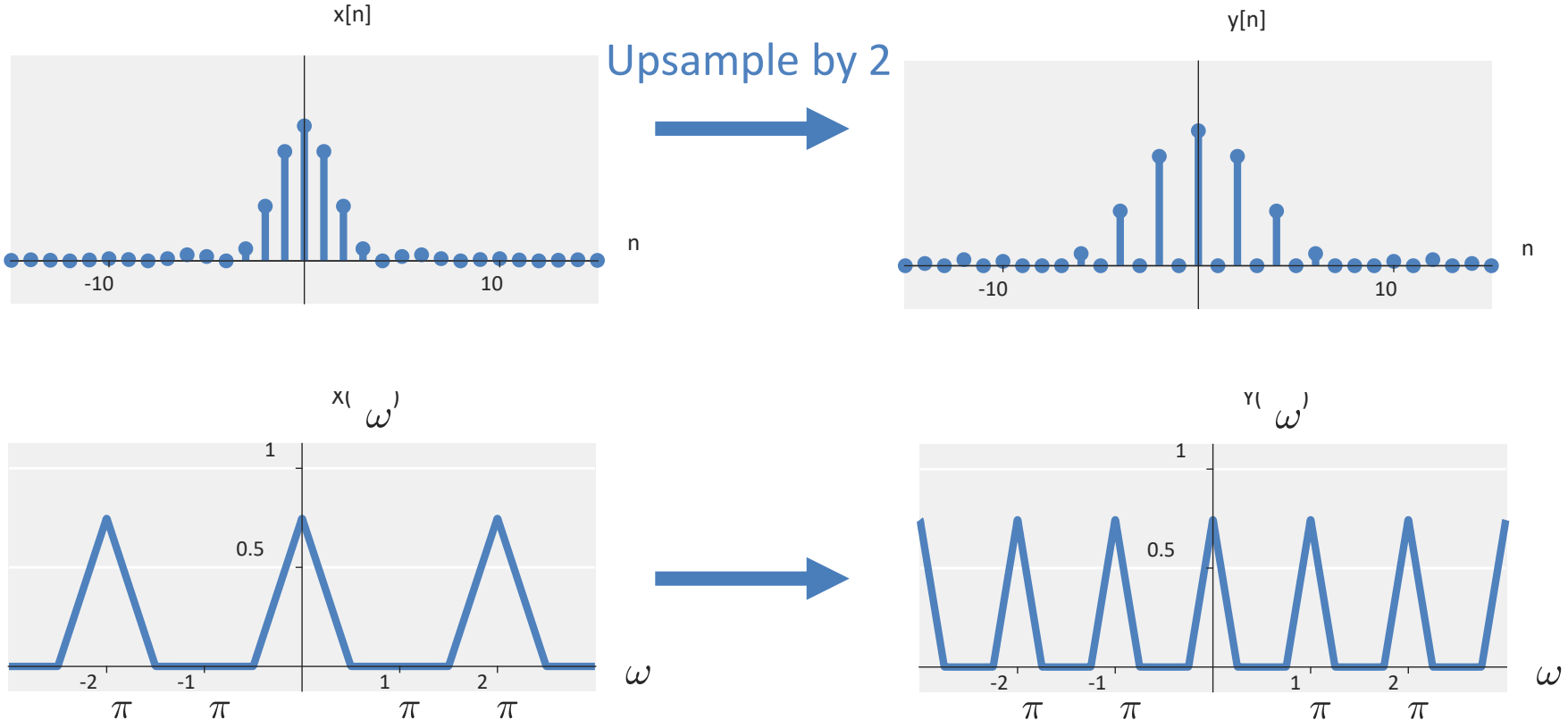


  - Shrink entire frequency-domain signal by N
  - Amplitude does not change

$$\mathcal{F}\{y[n]\} = \mathrm{X}(N\omega)$$

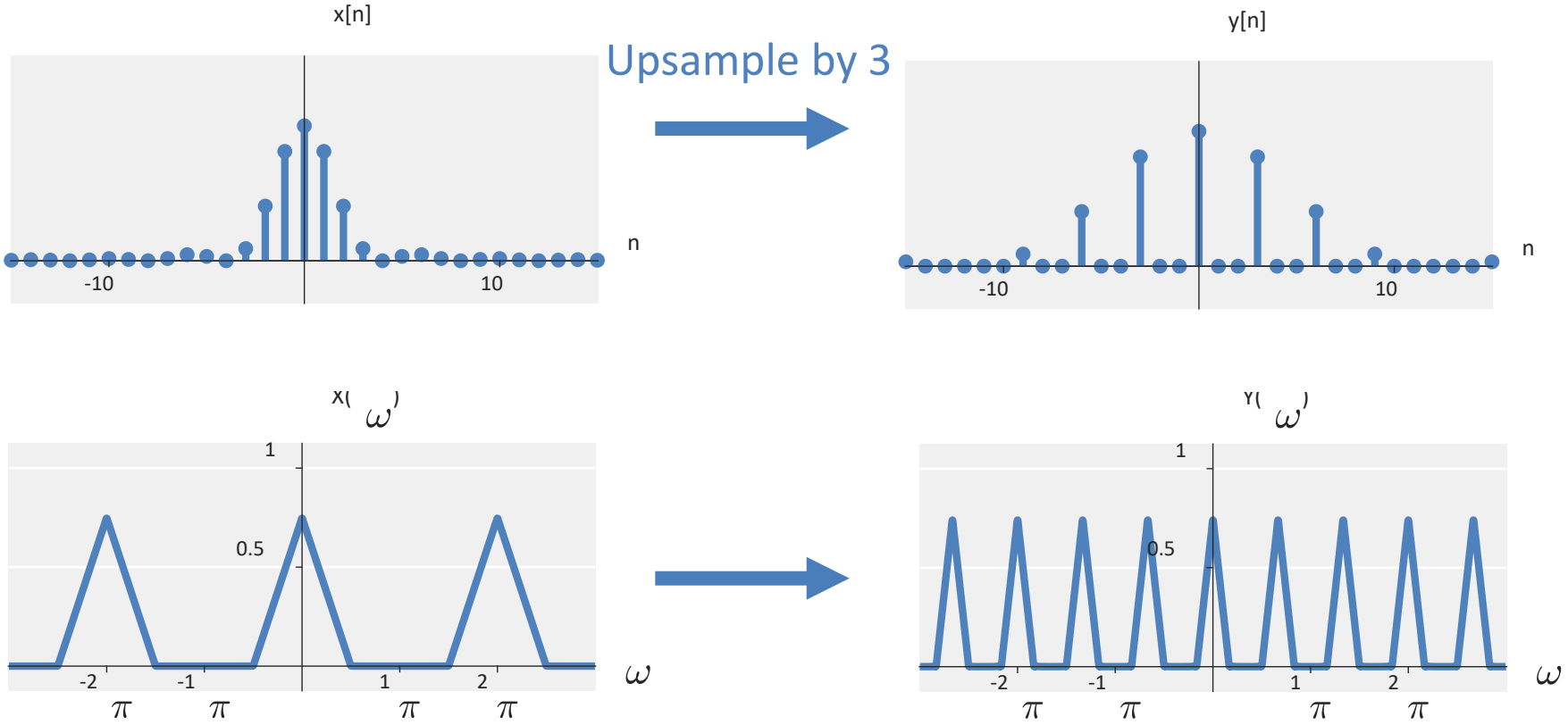
Upsampling

- **So what happens in frequency?**
  - Shrink entire frequency-domain signal by 2
  - Amplitude does not change



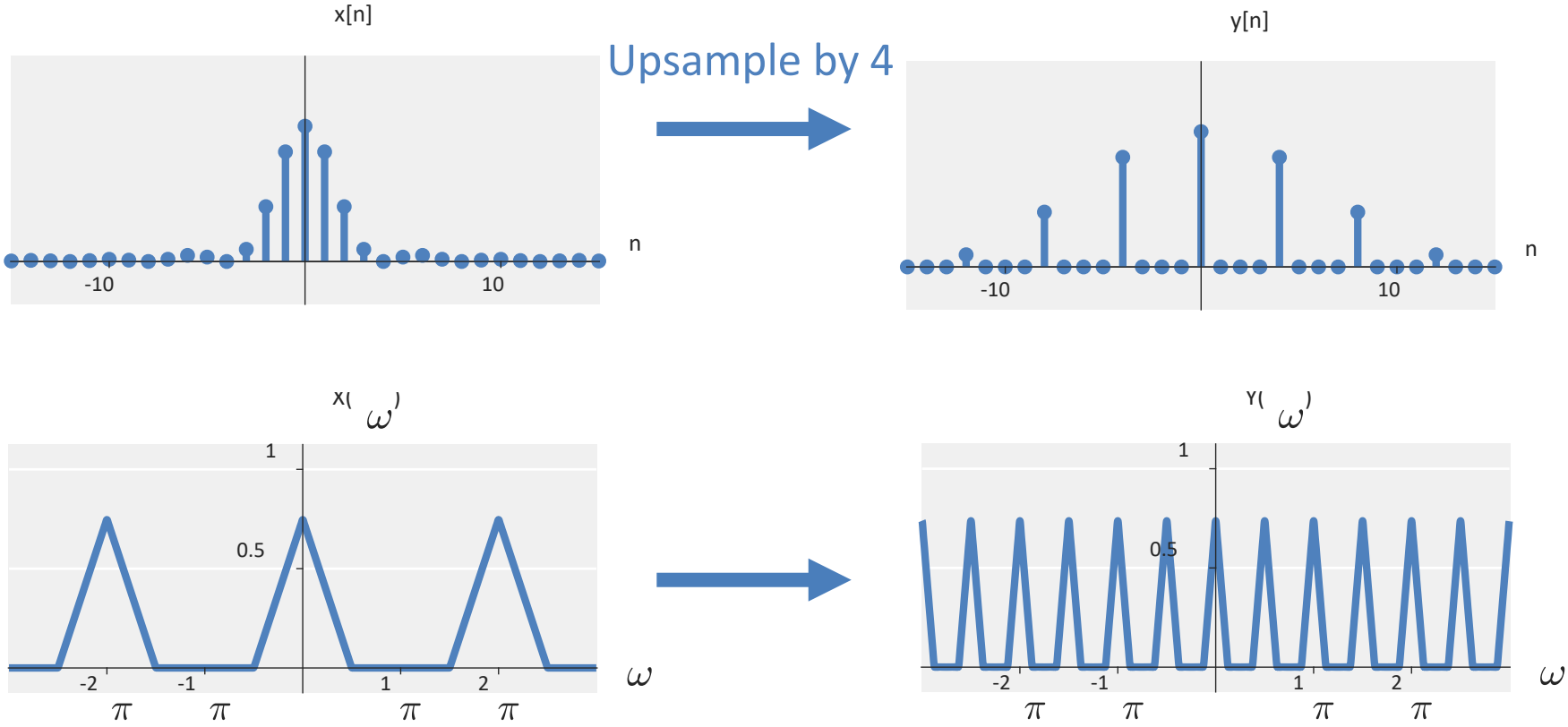
Upsampling

- **So what happens in frequency?**
  - Shrink entire frequency-domain signal by 3
  - Amplitude does not change



Upsampling

- **So what happens in frequency?**
  - Shrink entire frequency-domain signal by 4
  - Amplitude does not change



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Outline

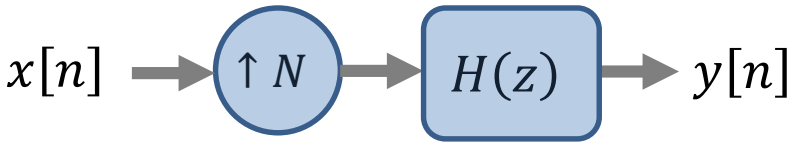
- Downsampling
- Downsampling with Anti-Aliasing Filter
- Upsampling
- **Upsampling with Reconstruction Filter**
- Resampling

Upsampling

- **Question: How do we get the higher sampling rate?**

Upsampling

- **Question: How do we get the higher sampling rate?**



Low pass filter

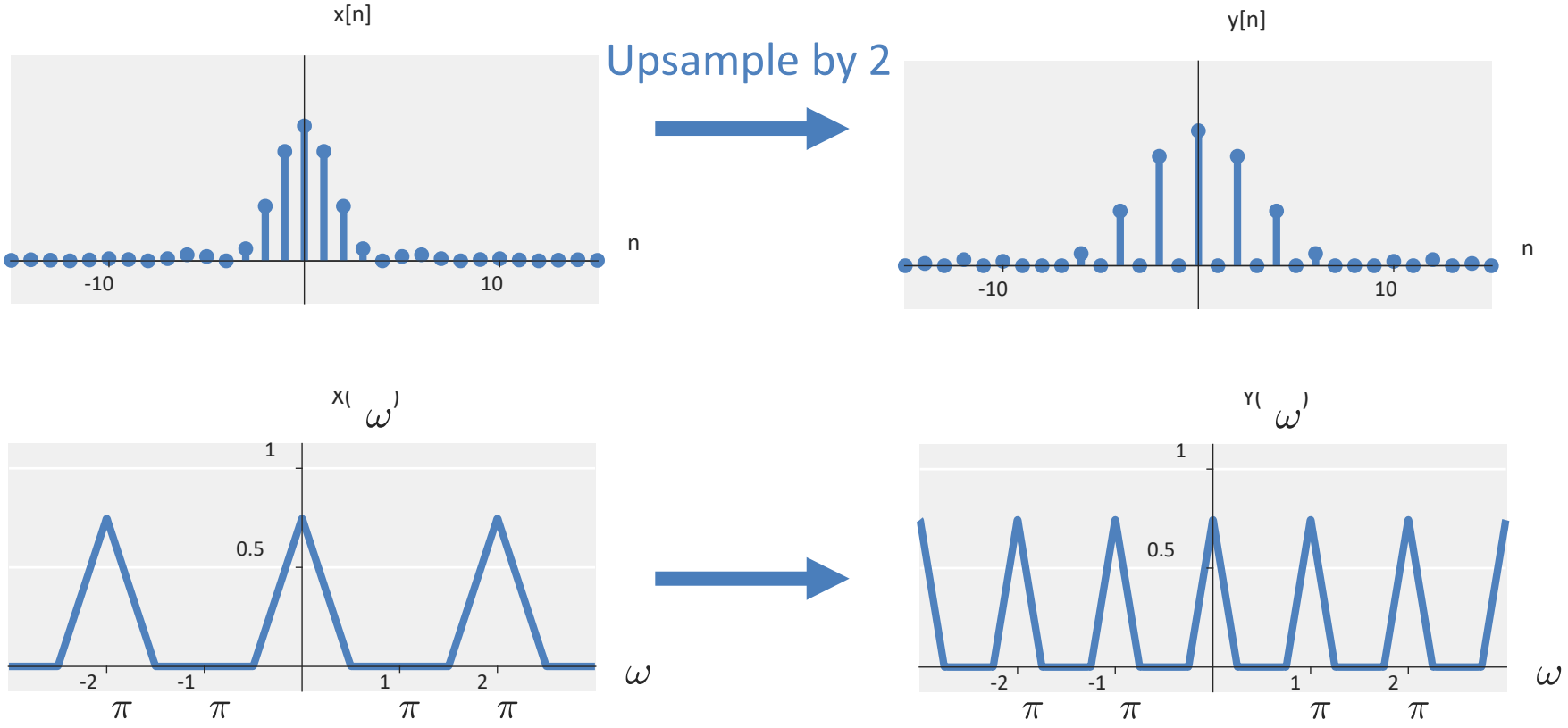
Cutoff: π/N

Gain: N

- Shrink entire frequency-domain signal by N
- Amplitude does not change
- Apply reconstruction filter

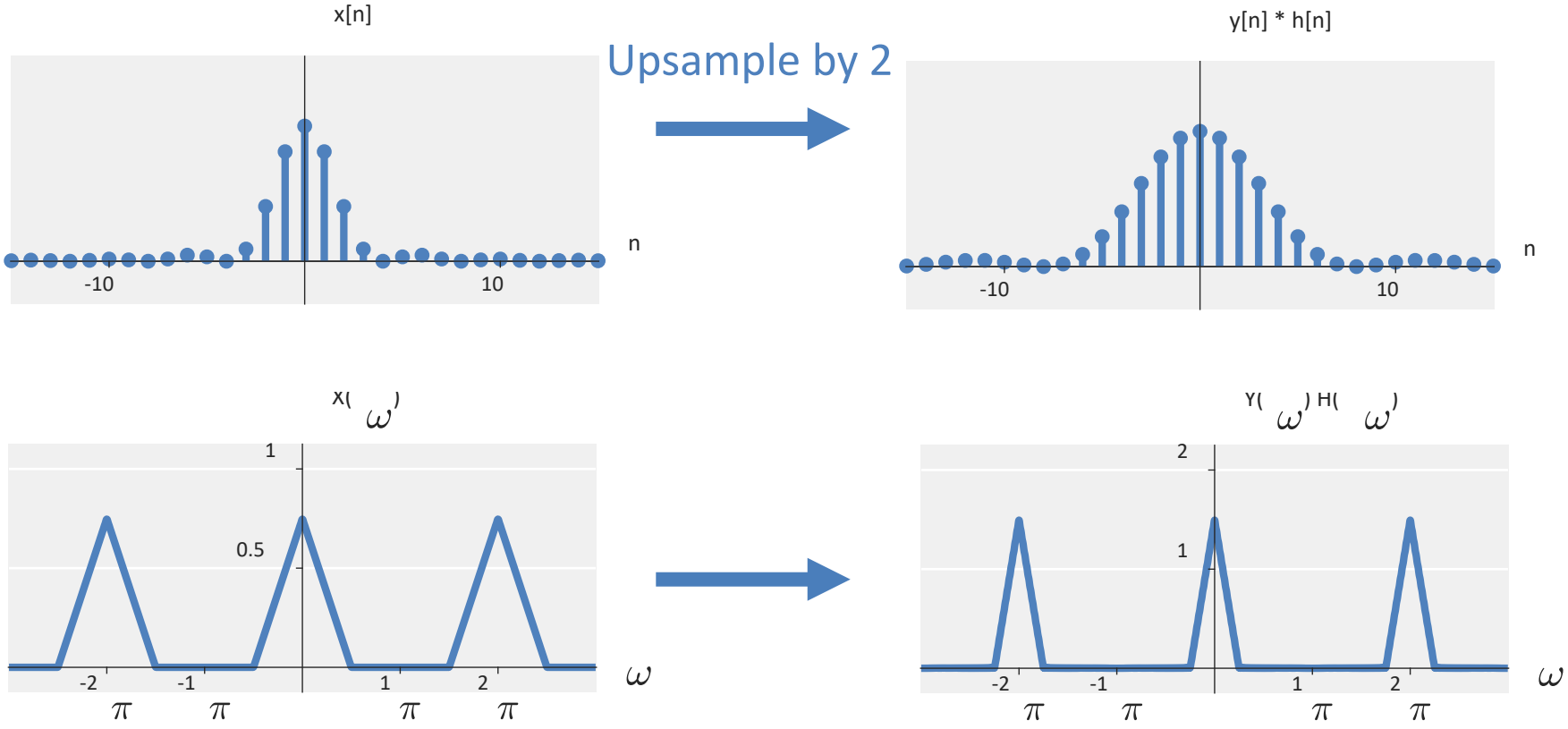
Upsampling

- **So what happens in frequency?**
  - Shrink entire frequency-domain signal by 2
  - Amplitude does not change



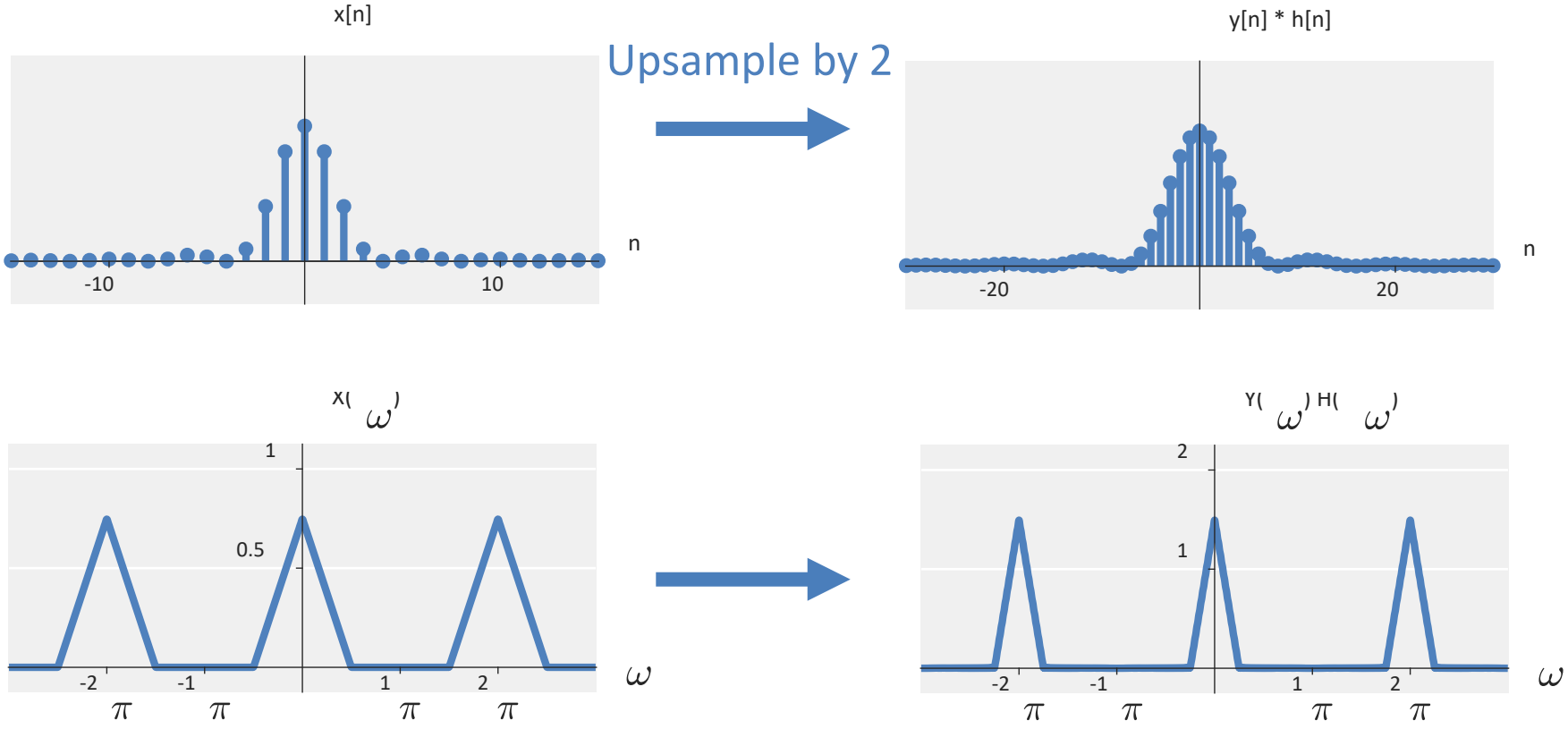
Upsampling

- **So what happens in frequency?**
  - Shrink entire frequency-domain signal by 2
  - Amplitude does not change



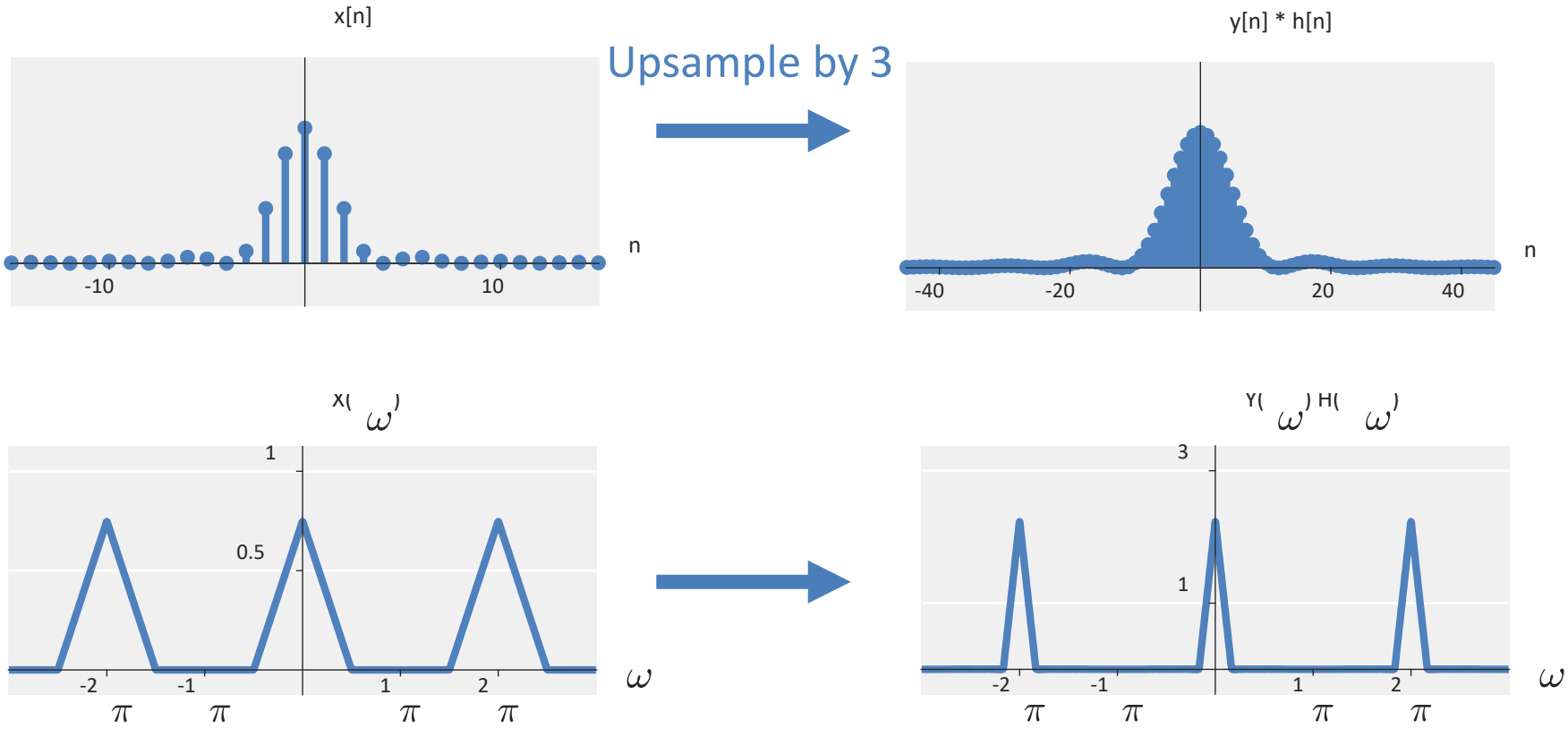
Upsampling

- **So what happens in frequency?**
  - Shrink entire frequency-domain signal by 2
  - Amplitude does not change



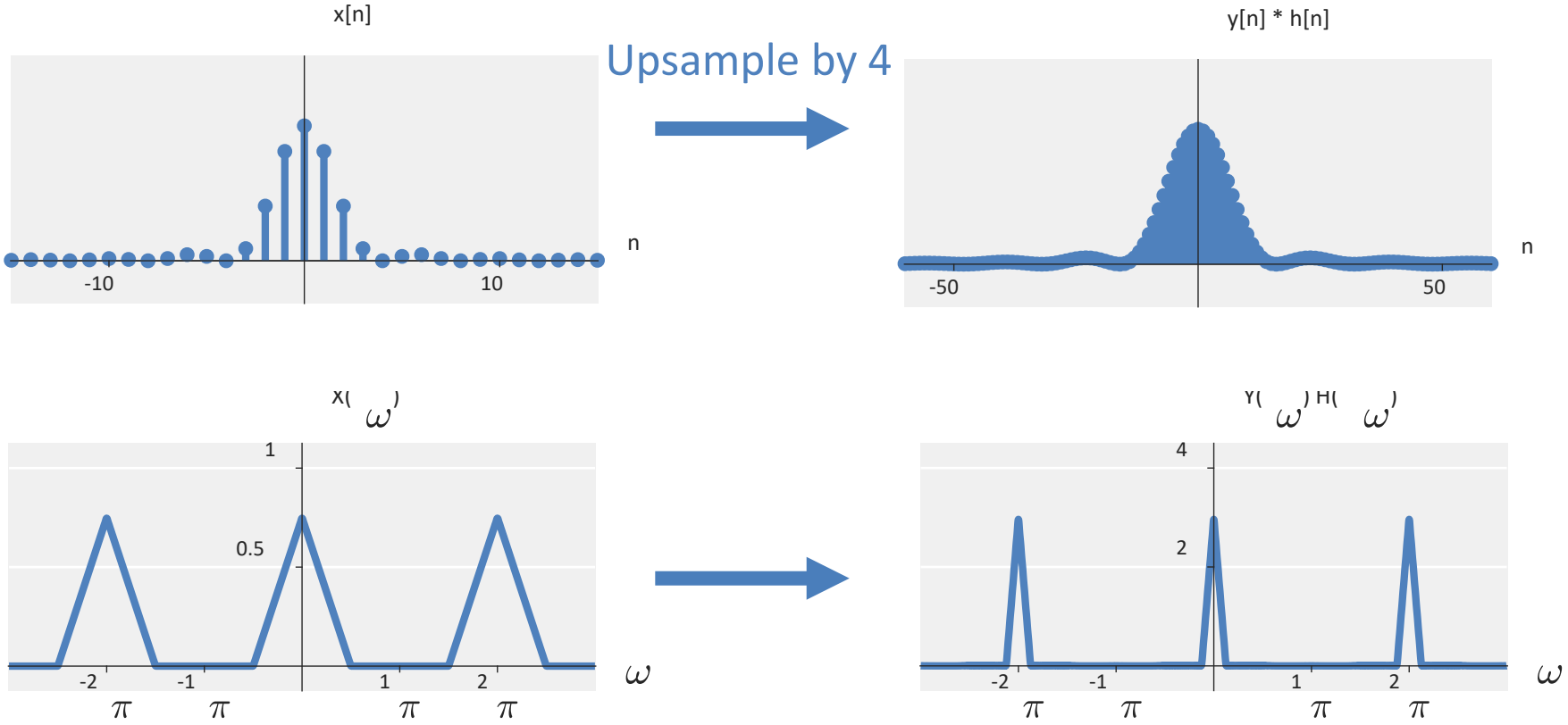
Upsampling

- **So what happens in frequency?**
  - Shrink entire frequency-domain signal by 3
  - Amplitude does not change



Upsampling

- **So what happens in frequency?**
  - Shrink entire frequency-domain signal by 4
  - Amplitude does not change



Lecture 24: Downsampling & Upsampling

Foundations of Digital Signal Processing

Outline

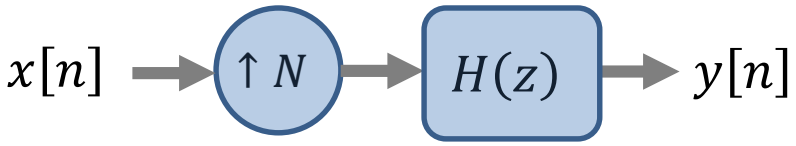
- Downsampling
- Downsampling with Anti-Aliasing Filter
- Upsampling
- Upsampling with Reconstruction Filter
- **Resampling**

Resampling

- **Question: How do non-integer sampling rate changes?**

Resampling

- **Question: How do non-integer sampling rate changes?**



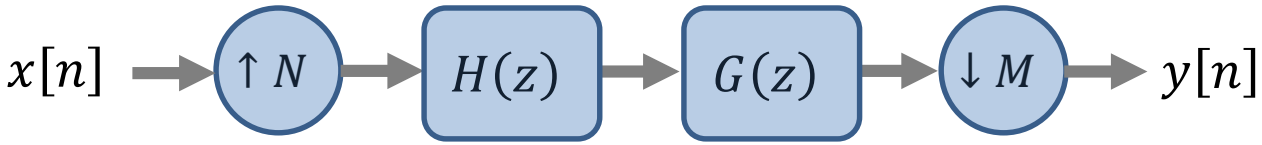
Low pass filter

Cutoff: π/N

Gain: N

Resampling

- **Question: How do non-integer sampling rate changes?**

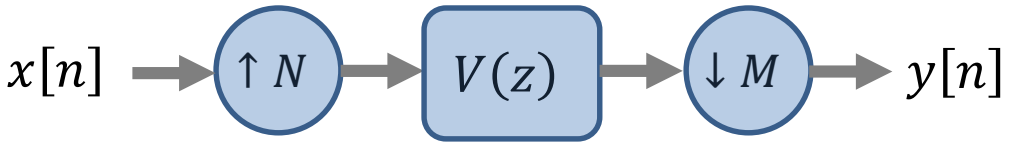


Low pass filter
Cutoff: π/N
Gain: N

Low pass filter
Cutoff: π/M
Gain: 1

Resampling

- **Question: How do non-integer sampling rate changes?**



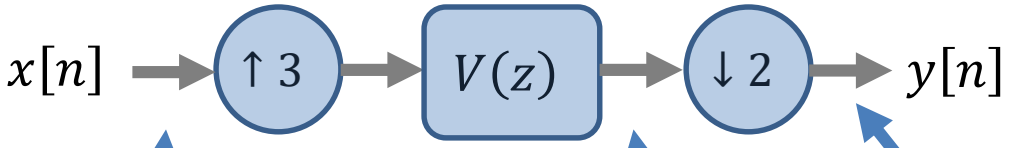
Low pass filter

Cutoff: $\min\left(\frac{\pi}{N}, \frac{\pi}{M}\right)$

Gain: N

Resampling

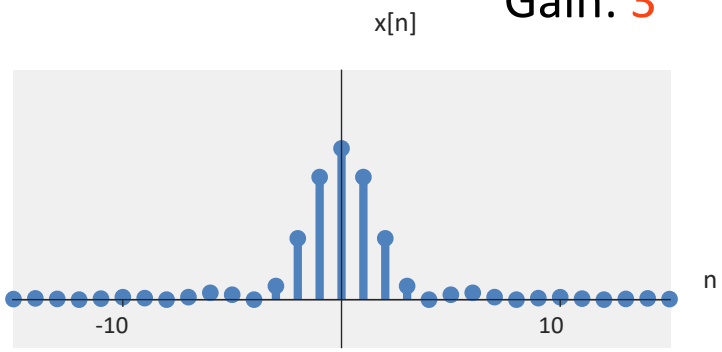
- **Question: How do non-integer sampling rate changes?**

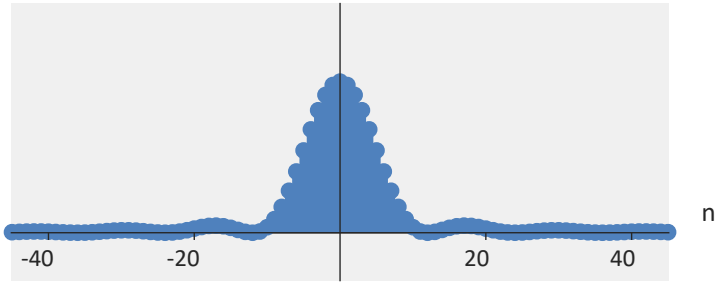


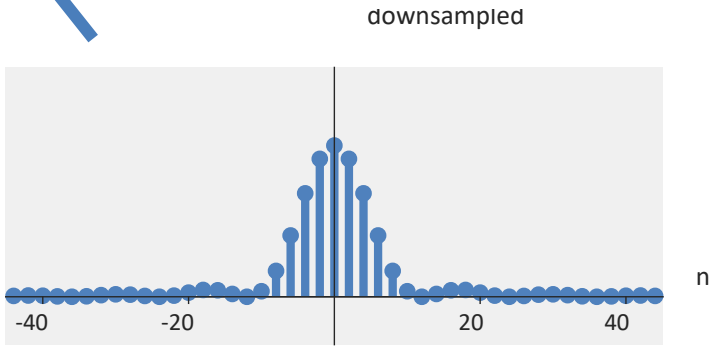
Low pass filter

Cutoff: $\frac{\pi}{3}$

Gain: 3

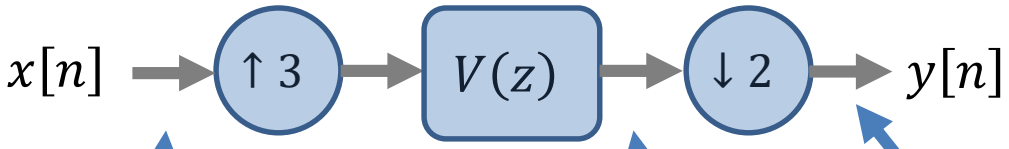






Resampling

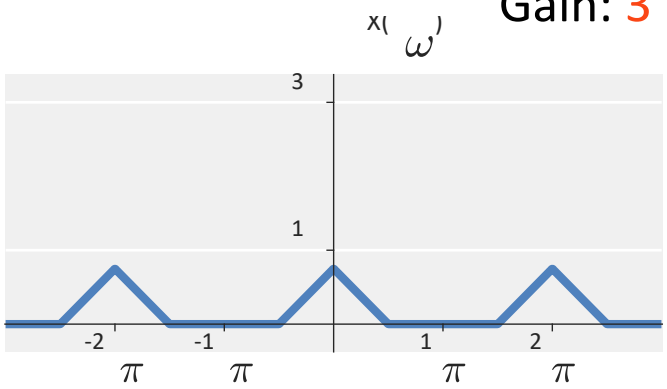
- **Question: How do non-integer sampling rate changes?**



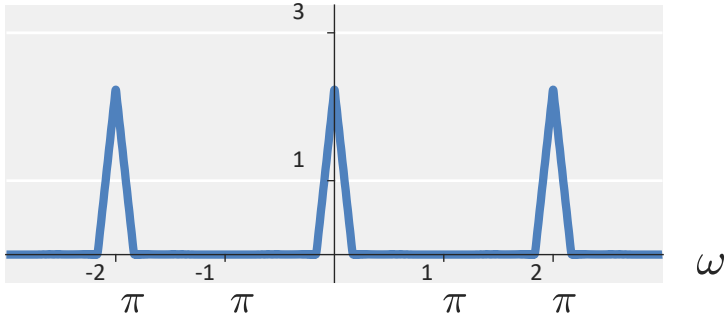
Low pass filter

Cutoff: $\frac{\pi}{3}$

Gain: 3



upsampled



downsampled