

Question	# of Points Possible	# of Points Obtained	Grader
# 1	17		
# 2	15		
# 3	16		
# 4	18		
# 5	18		
# 6	16		
Total	100		

**For full credit when sketching:** remember to label axes and make locations and amplitudes clear.

**Before starting the exam, read and sign the following agreement.**

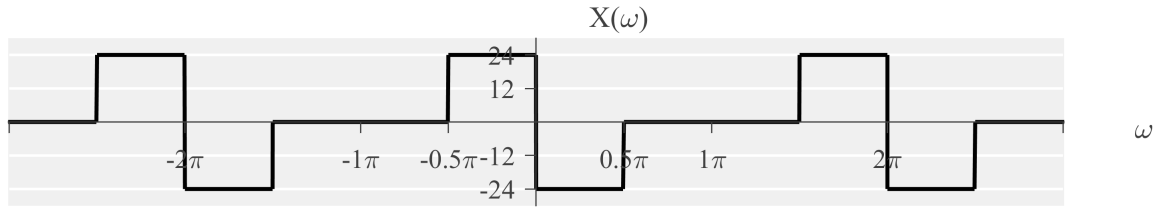
By signing this agreement, I agree to solve the problems of this exam while adhering to the policies and guidelines of the University of Florida and EEL 4750 / EEE 5502 and without additional external help. The guidelines include, but are not limited to,

- The University of Florida honor pledge: “On my honor, I have neither given nor received unauthorized aid in doing this assignment.”
- Only one 8.5 by 11 inch cheat sheet (double-sided) may be used
- No calculators or computers may be used
- No textbooks or additional notes may be used
- No collaboration is allowed
- No cheating is allowed

\_\_\_\_\_  
Student

\_\_\_\_\_  
Date

**Question #1:** Consider the DTFT of the signal  $x[n]$  (i.e.,  $X(\omega)$ ) shown below.

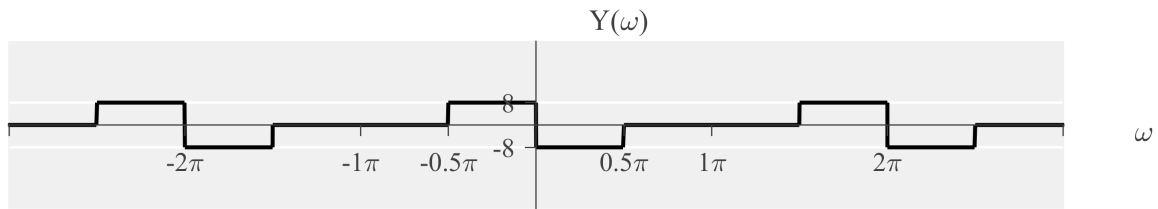


(a) (4 pts) What is the maximum achievable downsampling factor for  $5x[n]$  without aliasing?

**Solution:** The maximum downsampling factor is 2

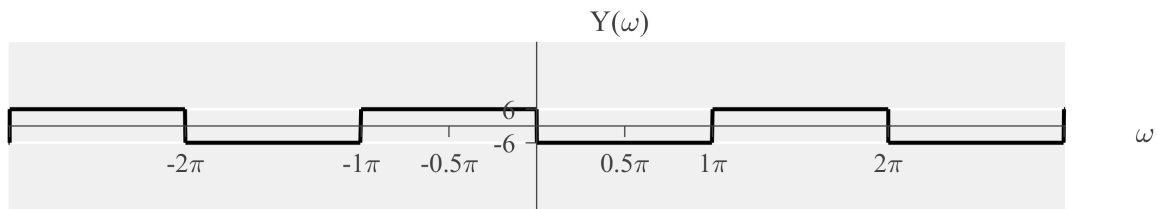
(b) (7 pts) Sketch the DTFT (from  $\omega = -3\pi$  to  $\omega = 3\pi$ ) of  $x[n]$  after downsampling by 3 (with no anti-aliasing filter). Remember to label important locations / values.

**Solution:**

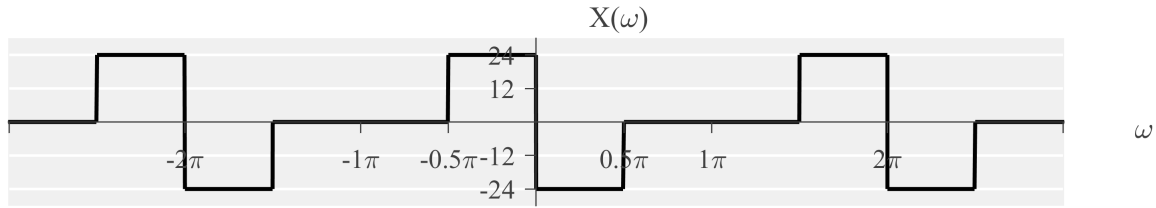


(c) (8 pts) Sketch the DTFT (from  $\omega = -3\pi$  to  $\omega = 3\pi$ ) of  $x[n]$  after downsampling by 4 (with an anti-aliasing filter). Remember to label important locations / values.

**Solution:**

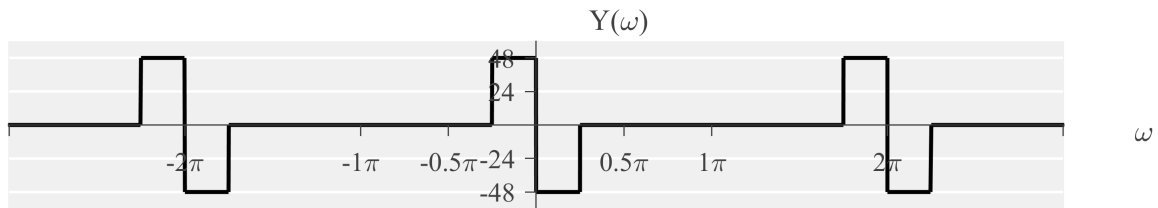


Question #2: Consider the DTFT of the signal  $x[n]$  (i.e.,  $X(\omega)$ ) shown below.



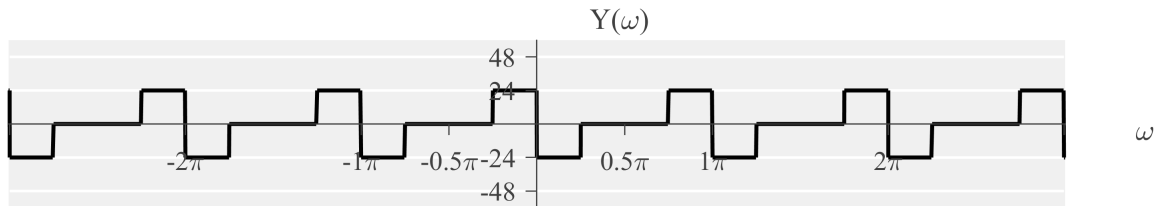
(a) (8 pts) Sketch the DTFT (from  $\omega = -3\pi$  to  $\omega = 3\pi$ ) of  $x[n]$  after upsampling by 2 (with an interpolation filter). Remember to label important locations / values.

**Solution:**

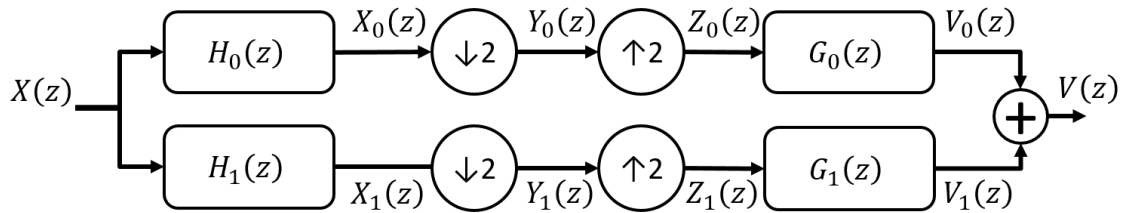


(b) (7 pts) Sketch the DTFT (from  $\omega = -3\pi$  to  $\omega = 3\pi$ ) of  $x[n]$  after upsampling by 2 (with no interpolation filter). Remember to label important locations / values.

**Solution:**



Question #3: Consider a 2-channel filter bank shown below.



Let the filters be defined by the frequency domain expression

$$H_0(\omega) = G_0(\omega) = \sqrt{2} \sin(\omega/2)$$

(a) (7 pts) Choose a filter  $H_1(\omega) = G_1(\omega)$  that satisfies the alias canceling conditions.

**Solution:** The alias canceling conditions:

$$\begin{aligned} H_0(\omega)G_0(\omega) + H_1(\omega)G_1(\omega) &= 2 \\ H_0(\omega - \pi)G_0(\omega) + H_1(\omega - \pi)G_1(\omega) &= 0 \end{aligned}$$

$$\begin{aligned} 2 \sin(\omega/2) \sin(\omega/2) + H_1(\omega)G_1(\omega) &= 2 \\ 2 \sin((\omega - \pi)/2) \sin(\omega/2) + H_1(\omega - \pi)G_1(\omega) &= 0 \end{aligned}$$

$$\begin{aligned} 2 \sin^2(\omega/2) + H_1(\omega)G_1(\omega) &= 2 \\ 2 \cos(\omega/2) \sin(\omega/2) + H_1(\omega - \pi)G_1(\omega) &= 0 \end{aligned}$$

If we choose  $H_1(\omega) = G_1(\omega) = \sqrt{2} \cos(\omega/2)$ ,

$$\begin{aligned} 2 \sin^2(\omega/2) + 2 \cos^2(\omega/2) &= 2 \\ 2 \cos(\omega/2) \sin(\omega/2) + 2 \cos((\omega - \pi)/2) \cos(\omega) &= 0 \end{aligned}$$

$$\begin{aligned} 2 \sin^2(\omega/2) + 2 \cos^2(\omega/2) &= 2 \\ 2 \cos(\omega/2) \sin(\omega/2) - 2 \sin(\omega/2) \cos(\omega) &= 0 \end{aligned}$$

$$2 = 2$$

$$0 = 0$$

(b) (7 pts) Let  $X(\omega) = \cos(\omega/2)$ . Compute the intermediate signal  $V_0(\omega)$ .

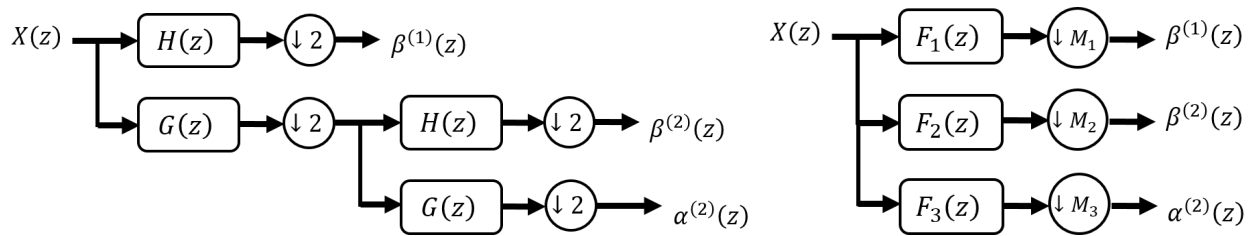
**Solution:** The frequency response at  $V_0(\omega)$  is

$$\begin{aligned} V_0(\omega) &= (1/2) [H_0(\omega)X(\omega) + H_0(\omega - \pi)X(\omega - \pi)] G_0(\omega) \\ &= (1/2) [\sin(\omega/2) \cos(\omega/2) + \sin((\omega - \pi)/2) \cos((\omega - \pi)/2)] G_0(\omega) \\ &= (1/2) [\sin(\omega/2) \cos(\omega/2) - \cos(\omega/2) \sin(\omega/2)] G_0(\omega) \\ &= 0 \end{aligned}$$

(c) (4 pts) (**True or False**) When the alias canceling conditions are met,  $V_0(z) = V_1(z)$ .

**Solution: False**, alias canceling ensures that  $V_0(z) + V_1(z) = X(z)$ , which is not guaranteed to be true when  $V_0(z) = V_1(z)$ .

**Question #4:** Consider the following wavelet bank and filter bank.



Let the high pass filter  $H(z)$  and low pass filter  $G(z)$  be defined by frequency responses:

$$G(\omega) = \sum_{k=-\infty}^{\infty} u(\omega + \pi/2 - 2\pi k) - u(\omega - \pi/2 - 2\pi k)$$

$$H(\omega) = \sum_{k=-\infty}^{\infty} u(\omega + \pi/2 - \pi - 2\pi k) - u(\omega - \pi/2 - \pi - 2\pi k)$$

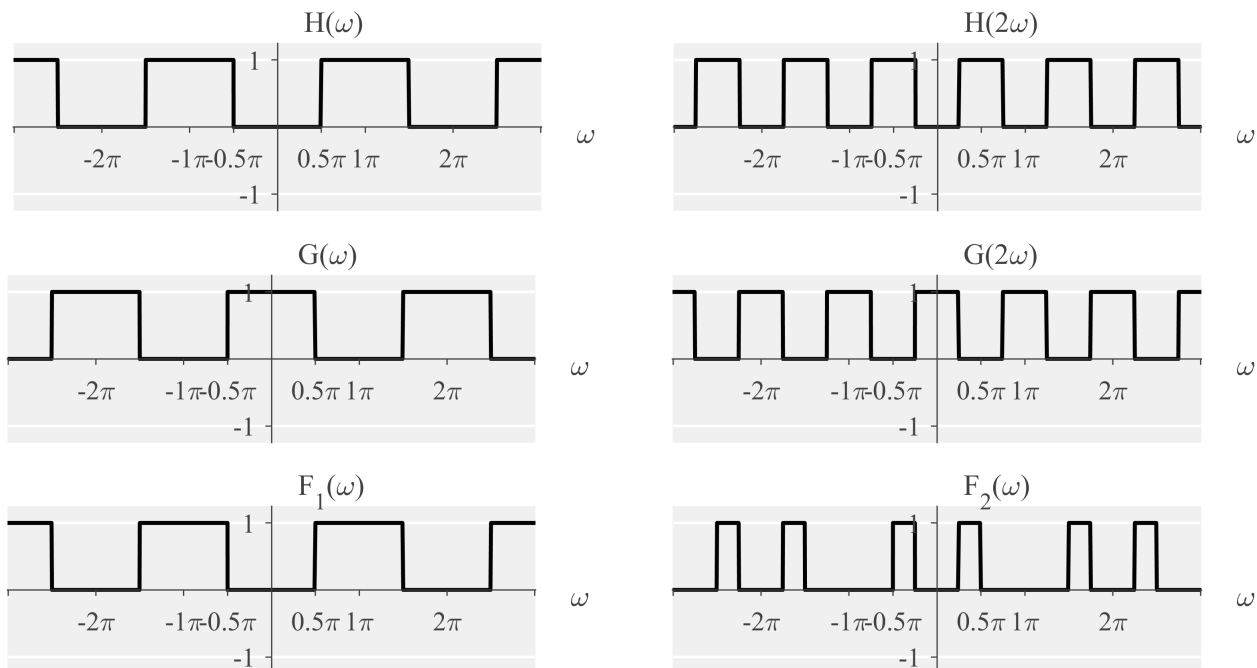
Use the Noble identities to simplify the wavelet bank (left) diagram and represent it as a filter bank (right). Determine  $M_1$ ,  $M_2$ , and  $M_3$ . Sketch  $|F_1(\omega)|$ ,  $|F_2(\omega)|$ , and  $|F_3(\omega)|$ .

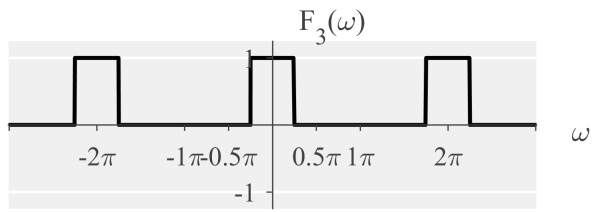
**Solution:**  $M_1 = 2$ ,  $M_2 = 4$ ,  $M_3 = 4$ .

$$F_1(\omega) = H(z)$$

$$F_2(\omega) = G(z)H(z^2)$$

$$F_3(\omega) = G(z)G(z^2)$$





**Table of Discrete-Time Fourier Transform Pairs:**

$$\text{Discrete-Time Fourier Transform} : X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$\text{Inverse Discrete-Time Fourier Transform} : x[n] = \frac{1}{2\pi} \int_{2\pi} X(\omega)e^{j\omega t} d\omega .$$

$x[n]$	$X(\omega)$	condition
$a^n u[n]$	$\frac{1}{1 - ae^{-j\omega}}$	$ a  < 1$
$(n+1)a^n u[n]$	$\frac{1}{(1 - ae^{-j\omega})^2}$	$ a  < 1$
$\frac{(n+r-1)!}{n!(r-1)!} a^n u[n]$	$\frac{1}{(1 - ae^{-j\omega})^r}$	$ a  < 1$
$\delta[n]$	1	
$\delta[n - n_0]$	$e^{-j\omega n_0}$	
$x[n] = 1$	$2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$	
$u[n]$	$\frac{1}{1 - e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi \delta(\omega - 2\pi k)$	
$e^{j\omega_0 n}$	$2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - \omega_0 - 2\pi k)$	
$\cos(\omega_0 n)$	$\pi \sum_{k=-\infty}^{\infty} \{\delta(\omega - \omega_0 - 2\pi k) + \delta(\omega + \omega_0 - 2\pi k)\}$	
$\sin(\omega_0 n)$	$\frac{\pi}{j} \sum_{k=-\infty}^{\infty} \{\delta(\omega - \omega_0 - 2\pi k) - \delta(\omega + \omega_0 - 2\pi k)\}$	
$\sum_{k=-\infty}^{\infty} \delta[n - kN]$	$\frac{2\pi}{N} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{N}\right)$	
$x[n] = \begin{cases} 1 & ,  n  \leq N \\ 0 & ,  n  > N \end{cases}$	$\frac{\sin(\omega(N+1/2))}{\sin(\omega/2)}$	
$\frac{\sin(Wn)}{\pi n} = \frac{W}{\pi} \text{sinc}\left(\frac{Wn}{\pi}\right)$	$X(\omega) = \begin{cases} 1 & , 0 \leq  \omega  \leq W \\ 0 & , W <  \omega  \leq \pi \end{cases}$	
$X(\omega)$ is periodic with period $2\pi$		



**Table of Discrete-Time Fourier Transform Properties:** For each property, assume

$$x[n] \xleftrightarrow{DTFT} X(\omega) \quad \text{and} \quad y[n] \xleftrightarrow{DTFT} Y(\omega)$$

Property	Time domain	DTFT domain
Linearity	$Ax[n] + By[n]$	$AX(\omega) + BY(\omega)$
Time Shifting	$x[n - n_0]$	$X(\omega)e^{-j\omega n_0}$
Frequency Shifting	$x[n]e^{j\omega_0 n}$	$X(\omega - \omega_0)$
Conjugation	$x^*[n]$	$X^*(-\omega)$
Time Reversal	$x[-n]$	$X(-\omega)$
Convolution	$x[n] * y[n]$	$X(\omega)Y(\omega)$
Multiplication	$x[n]y[n]$	$\frac{1}{2\pi} \int_{2\pi} X(\theta)Y(\omega - \theta)d\theta$
Differencing in Time	$x[n] - x[n - 1]$	$(1 - e^{-j\omega})X(\omega)$
Accumulation	$\sum_{k=-\infty}^{\infty} x[k]$	$\frac{1}{1 - e^{-j\omega}} + \pi X(0) \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$
Frequency Differentiation	$nx[n]$	$j \frac{dX(\omega)}{d\omega}$
Parseval's Relation for Aperiodic Signals	$\sum_{k=-\infty}^{\infty}  x[k] ^2$	$\frac{1}{2\pi} \int_{2\pi}  X(\omega) ^2 d\omega$

**Table of Z-Transform Pairs:**

$$\text{Z-Transform} : X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$$\text{Inverse Z-Transform} : x[n] = \frac{1}{2\pi j} \oint_{\mathcal{C}} X(z)z^{n-1} dz .$$

$x[n]$	$X(\omega)$	ROC
$a^n u[n]$	$\frac{1}{1 - az^{-1}}$	$ z  >  a $
$-a^n u[-n - 1]$	$\frac{1}{1 - az^{-1}}$	$ z  <  a $
$na^n u[n]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$ z  >  a $
$-na^n u[-n - 1]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$ z  <  a $
$\delta[n]$	1	All $z$
$\delta[n - n_0]$	$z^{-n_0}$	All $z$
$u[n]$	$\frac{1}{1 - z^{-1}}$	$ z  > 1$
$\cos(\omega_0 n)u[n]$	$\frac{1 - z^{-1} \cos(\omega_0)}{1 - 2z^{-1} \cos(\omega_0) + z^{-2}}$	$ z  > 1$
$\sin(\omega_0 n)u[n]$	$\frac{z^{-1} \sin(\omega_0)}{1 - 2z^{-1} \cos(\omega_0) + z^{-2}}$	$ z  > 1$
$a^n \cos(\omega_0 n)u[n]$	$\frac{1 - az^{-1} \cos(\omega_0)}{1 - 2az^{-1} \cos(\omega_0) + a^2 z^{-2}}$	$ z  >  a $
$a^n \sin(\omega_0 n)u[n]$	$\frac{az^{-1} \sin(\omega_0)}{1 - 2az^{-1} \cos(\omega_0) + a^2 z^{-2}}$	$ z  >  a $

**Table of Z-Transform Properties:** For each property, assume

$$x[n] \xleftrightarrow{Z} X(z) \quad \text{and} \quad y[n] \xleftrightarrow{Z} Y(z)$$

Property	Time domain	Z-domain
Linearity	$Ax[n] + By[n]$	$AX(z) + BY(z)$
Time Shifting	$x[n - n_0]$	$X(z)z^{-n_0}$
Z-scaling	$a^n x[n]$	$X(a^{-1}z)$
Conjugation	$x^*[n]$	$X^*(z^*)$
Time Reversal	$x[-n]$	$X(z^{-1})$
Convolution	$x[n] * y[n]$	$X(z)Y(z)$
Differentiation in z-domain	$nx[n]$	$-z \frac{dX(z)}{dz}$
Initial Value Theorem	$x[n]$ is causal	$x(0) = \lim_{z \rightarrow \infty} X(z)$